REPORT No. 201

GEOLOGICAL SURVEY OF JAPAN

ON THE NEW METHOD OF ANALYSIS IN GRAVITY PROSPECTING

By
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Masatsugu SAITO, Director

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CONTENTS

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Abstract

There are two problems in the analysis of gravity anomalies, namely, one is to detect local and weak anomalies and the other is to interpret quantitatively those. In the former problem there are many methods, but in the latter problem the established method is not present.

In the present paper, the results of writer's study on these problems are mentioned. That is, at first he considered in detail the filtering effects of the detections of the "Running Average Method" which was previously proposed by him as the method to detect local and weak gravity anomalies, and secondly he described the method by which subterranean density anomalies(residual densities)were directly calculated from the residual gravities obtained by his method.

In the former considerations, he always paid attention to the possibility of applications to other geophysical prospectings. And he could have the very important and interesting results. In the latter considerations, he considered particularly the filtering action in detail which the equation of the Fourier transform representation of the residual density—residual gravity relation meant. Then as the results of this consideration very interesting and important results were obtained.

I. Introduction

Bouguer anomaly observed on the earth surface is caused by the subterranean mass distribution. Then, we can presume the geological structures from the Bouguer anomaly distribution qualitatively. But in this interpretation of gravity we must always pay attention to that subterranean mass distribution can not be calculated theoretically from the Bouguer anomaly distribution without any assumptions on the subterranean density distribution. In other words, as the gravity method based upon the potential theory, it is difficult to obtain the unique solution of the subterranean density distribution from the Bouguer anomalies. However, besides gravity data, usually in many cases we have various data, those are geological, bore hole and seismic data, etc., consequently subterranean structures, which can be presumed, are very restricted by using these various data. For instance, from these various data in many surveys we consider that observed values of gravity are composed of regional gravity which may be caused by bedrock and local gravity which may be caused by undulation of basement or subterranean structures of various scales at any depth. And on the other hand, we can assume that the density increases with depth. From this point of view, the sectional calculation of a basement has often been performed by using of the trial and error method or the direct method in two dimensional approximations. And for local gravity, we have often estimated its depth and calculated roughly its shape assuming the existence of the massive body. However, in the case of presence of the geological structures of comparatively small scale, it is difficult even to point out the presence of these geological structures, and still more it is very difficult to estimate the depth of these geological structures and to presume their shapes. Therefore, it is necessary to consider the following two points to solve these difficult pro-

blems.
(1) To detect local gravities which may be caused by geological structures.
(2) To know the distribution of subterranean density anomalies by assuming the adequate model of density distribution from the corresponding gravity anomalies detected by the method obtained in the investigation of the subject (1).

In the case of the surveys which objects are to point out the presence of some geological structures of comparatively small scale and to presume their kinds and to estimate their depths, the detection of a weak and local gravity anomaly and its interpretation have great significance. Then in the following, outlines of the previous and the writer's studies on the above two subjects will be described.

As the method to detect a weak anomaly of objective scale, the various methods were proposed in the past. These methods can be classified into two methods, i. e., one is the residual gravity method and the other is the derivative method. For a while, these two methods will be explained very briefly. Then after, the outline of the writer's study on the subject (1) will be mentioned.

Generally speaking, the strong point of the residual gravity method is the simple interpretation of the residuals compared with the latter method. In the former, there are many methods, those are (1) profile method, (2) smoothed contour method, (3) Fourier analysis method, (4) Griffin's method [9], (5) method of least squares (Agocs[1], Simpson[24], Oldham and Sutherland[17]) and (6) minimum variance method (Brown[3]).

In these methods, the first two are the simplest methods, but a residual gravity obtained can not be considered theoretically and a weak anomaly can not be detected by these methods, then in the presence these methods are not used. The method (3) seems to be the exact method apparently, but this is not sufficiently analyzed by reason of both the natures of compensation and the structural character[21] in which any terms of the Fourier series have not been taken into consider ation. And further, the calculation in this method is complicated, so this method is not performed generally. The method (4) is the simple method and has the analytical meaning (i. e. this method is recognized as the 1st approximation of the 2nd derivative method), therefore, this method is often used in surveys. However, it is supposed that in this method an influence of noise is large. The both methods of the last are the methods to obtain a regional gravity, but the calculations by these methods are very complicated and then practically the calculations are carried out by using the electronic computer. However, the use of this method gives us no significant information despite of the labor of calculation.

Now, here in the derivative method there are two kinds, i. e., (1) the 1st vertical gradient method and (2) the 2nd derivative method. In the former there are the formulae proposed by Evjen [7], Tsuboi[29] and Kato[13], but in these formulae Evjen's and Tsuboi's formulae include the treatment of the integral, therefore, these formulae are not suitable for the practical use. A vertical gradient is a quantity concerning the free air reduction and then its physical meaning is important. However, in the case of interpretation of the gravity anomaly it becomes difficult to presume the subterranean excess mass distribution from the distribution of the 1st vertical derivatives. Moreover, in this method an influence of noise is large too.

In the usual method of the gravity analysis there is the 2nd derivative method, and in the formulae of this method there are many formulae proposed by Peters[18], Henderson and Zietz [11], Elkins[6], Rosenbach [19] and Kato [13] and others. However, as pointed out by Kato[13], the 2nd vertical derivative has not the direct physical meaning as the 1st vertical derivative has, and as pointed out by Elkins[6], the errors caused by noises can become the same order magnitude as the 2nd derivatives. And then the interpretation of the gravity anomaly becomes more difficult by using this method than the case of the 1st vertical derivative method. Therefore, a new method to obtain a residual gravity is required.

The defects which are included in the previous various methods to detect the local weak anomaly are mentioned above. The desirable characters as the method to detect the weak anomaly are as follows:

(a) The calculations must be simply performed.
(b) The regional gravity and noises must be well removed.
(c) The anomaly of objective scale must be clearly pointed out.
(d) The anomalies obtained must become an object of the theoretical considerations.
(e) The anomalies obtained must have simple physical meaning.
(f) The interpretation of the gravity anomaly must be performed comparatively easily without high physical and mathematical knowledges.

The writer researched the method of gravity analysis possessing the characters mentioned above, and published "The Running Average Method"[21] which is a kind of the residual gravity method. This new method possesses the characters mentioned above and further other excellent characters. And so now his method is being adopted by our Geological Survey instead of the previous method (Henderson-Zietz's method).

"The Running Average Method" can be also used in other geophysical prospecting by reason of its excellent character. For example, the writer applied this method to detect the anomalous potential in the study of the spontaneous potential at the Oage Pyrite Mine[23] which area is covered by silicified rock, and he made success to detect a part of anomalous potentials which may be caused by ore deposits removing the characteristic potential of silicified zone and noise potentials. As the result of this study, it is found that anomalous potentials detected appear over the three of five known ore deposits.

In the present paper, at first the writer discusses the filtering effect of "running average method" in chapter II, and in chapter III he discusses the extension of this method to the two dimensional problems. In chapters II and III, he pays attention to the possibility of the applications to other geophysical prospecting. In chapter IV he considered the physical meanings of a quantity detected by the present method.

For the subject (2) we often perform the quantitative interpretation of gravity anomalies by the use of the adequate assumptions for the subterranean density distribution. That is, for the two dimensional cases the shape of the surface of the basement is calculated by means of the trial and error method or the direct method from the regional gravity, and so sometimes good results were obtained. And also for strong and local gravity anomalies which are caused by the existence of the massive bodies of large density the results of calculations in which the simple geometrical models are assumed for these bodies were often good. But when the residual gravity is caused by the geological structures within the very thick formations, its quantitative interpretation becomes very difficult.

The relation between the residual gravity and the subterranean excess mass distribution has being studied by many investigators. And in these studies the depth estimation of the excess mass was researched by the investigators who are Fisher[8] Kogbetliants[14], Saxov and Nygaard[20], Bott and Smith[2], Smith[25], etc. And then the estimation of the excess mass distribution at given depth was researched by Tsuboi and Fuchida[28], Hammer[10], Bullard and Cooper[5], etc. However, these studies assumed the massive distribution of the matter or the presence of the condensation surface. Then from each point of view of these studies it is supposed that it is difficult to interpret quantitatively a residual gravity for the geological strutures within the very thick formations.

The writer investigates the problem (2) successively to the proposal of the running average method.

In chapter VI, he discusses the method to obtain the subterranean excess mass distribution directly from the residual gravity by assuming the suitable mathematical model of the density distribution. In this method "the depth of the presence" of the subterranean excess mass is able to vary with the value of the parameter "k". This method can be also applied to the interpretation of the regional gravity. But in this case we can not well forecast analytically the excess mass distribution as in the case of the residual gravity of small scale. The consideration of the characteristics of "the density-spacial filter" in the relation between the distribution of the residual gravity and the distribution of the corresponding "residual density" is done in VI. 2, and the computing formulae of the residual density are obtained in VI. 3.

Acknowledgments

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II. Theoretical Consideration of the Running Average Method

II. 1 Definition of the Running Average Method

The technical term of "the running average method" has been named by the writer in the reference (6). And this is the inclusive name of the method to calculate the anomalous quantities of any scale at arbitrary point P_i set up with the same interval S on the x-axis, and this anomaly is calculated by subtracting the average value (arithmetic mean) of observed values at $(2\beta+1)$ points which center point is P_i from the average value of observed values at $(2\alpha+1)$ points with the same center point P_i. This method is illustrated in Fig. 1.

Now, when the position of the measurement point P_i is denoted by the symbol x_i ($i=0, 1, 2, 3, \cdots\cdots$), the observed value at the point P_i is denoted by the symbol g_i and when the anomalous quantity obtained by the present method at the same point P_i is denoted by $\triangle_{\alpha,\beta}\, g\,(x_i)$, this anomaly can be expressed by

$$\begin{aligned}\triangle_{\alpha,\beta}\, g(x_i) &= \frac{1}{(2\alpha+1)}\sum_{k=-\alpha}^{\alpha} g_{i+k} - \frac{1}{(2\beta+1)}\sum_{k=-\beta}^{\beta} g_{i+k} \\ &= -\frac{1}{(2\beta+1)}\left\{g_{-\beta+i} + g_{-\beta+i+1} + \cdots\cdots + g_{-\alpha+i-1}\right\} \\ &\quad + \frac{2\,(\beta-\alpha)}{(2\alpha+1)\,(2\beta+1)}\left\{g_{-\alpha+i} + \cdots\cdots + g_i + \cdots\cdots + g_{\alpha+i}\right\} \\ &\quad - \frac{1}{(2\beta+1)}\left\{g_{\alpha+i+1} + g_{\alpha+i+2} + \cdots\cdots + g_{\beta+i}\right\} \qquad (1)\end{aligned}$$

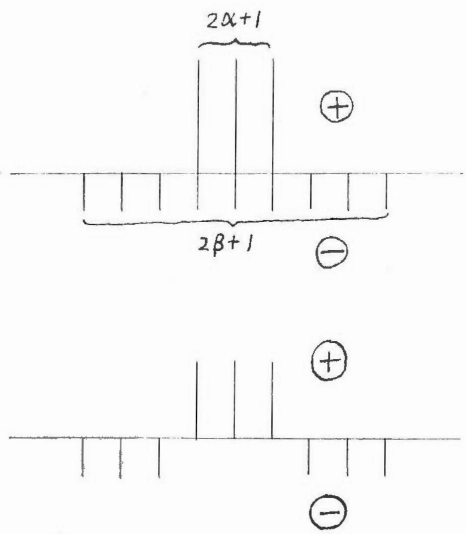

Fig. 1 Illustration of the running average method

Therefore, the present method which is able to understand as the operation of computing the difference between two arithmetic running averages each other differs is also understood as the operation of a kind of the weighted running average. Then the present method has been named "the running average method" because of the meaning mentioned above.

It is supposed that we can not always expect to obtain good results for arbitrary values of α and β in the present method. Then, in the reference (6), the writer investigates the filtering effects of the detections with the various values of α and β. As the result of this investigation, he arrived the conclusion that the combination of ($\alpha=1$, $\beta=3$) which has not necessarily best characteristic, but it should be usually used. Then after, he named "the normal detection" for this detection. Moreover, for the interpretation of the gravity anomaly he concluded that the both detections of ($\alpha=0$, $\beta=1$) and ($\alpha=3$, $\beta=7$) should be performed, and he named "the noise detection" and "the bistructural detection" for each detection respectively.

In the below sections the characteristics of the running average method as a sampling operation will be discussed.

II. 2 Filtering Effect of "the Extended Running Average Method"

In this section, the continuous sampling operation will be considered, which is the limiting case of the present method in order to investigate the analytical character of the present method as the selective sampling operation.

Now, suppose the physical quantity g is the function of (time or distance) variable x, then g is expressed by

$$g=g(x).$$

And if $g(x)$ has the continuous spectrum denoted by $G(\omega)$, where $\omega/2\pi$ is the frequency in cycles per unit time or the frequency in cycles per unit length in the

x-direction, $g(x)$ is expressed by

$$g(x)=\int_{-\infty}^{\infty} G(\omega)e^{i\omega x}d\omega. \tag{2}$$

And $G(\omega)$ is given by the Fourier transform of

$$G(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} g(x)e^{-i\omega x}dx. \tag{3}$$

Then $g(x)$ can also be expressed by

$$g(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi)e^{i\omega(x-\xi)}d\xi d\omega. \tag{4}$$

Here, when the average value of the quantity $g(x)$ in arbitrary range, $x_i-a=x=x_i+a$, is denoted by $\bar{g}^a(x_i)$, we have

$$\bar{g}^a(x_i)=\frac{1}{2a}\int_{x_i-a}^{x_i+a} g(x)dx$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi)\frac{\sin a\omega}{a\omega}e^{i\omega(x_i-\xi)}d\xi d\omega. \tag{5}$$

Similarly, the mean $\bar{g}^b(x_i)$ is given by

$$\bar{g}^b(x_i)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi)\frac{\sin b\omega}{b\omega}e^{i\omega(x_i-\xi)}d\xi\, d\omega. \tag{5$'$}$$

When the difference between above two means is denoted by $\Delta_{a,b}g(x)$, i. e.,

$$\Delta_{a,b}g(x)=\bar{g}^a(x)-\bar{g}^b(x),$$

the following Fourier integral representation is obtained for this quantity.

$$\Delta_{a,b}g(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_{a,b}(\omega)g(\xi)e^{i\omega(x-\xi)}d\xi d\omega. \tag{6}$$

where

$$K_{a,b}(\omega)=\frac{\sin a\omega}{a\omega}-\frac{\sin b\omega}{b\omega} \tag{7}$$

and when b is n times of a, another symbol $K_n^a(\omega)$ is used instead of $K_{a,b}(\omega)$, i. e.,

$$K_n^a(\omega)=\frac{1}{na\omega}\left\{n\sin a\omega-\sin na\omega\right\}. \tag{7$'$}$$

Eq. (6) is also expressed by

$$\Delta_{a,b}g(x)=\int_{-\infty}^{\infty} K_{a,b}(\omega)G(\omega)e^{i\omega x}d\omega. \tag{8}$$

Therefore, if the function $\Delta_{a,b}G(\omega)$ is the Fourier transform of the function $\Delta_{a,b}g(x)$, then the following frequency equation is obtained.

$$\Delta_{a,b}G(\omega)=K_{a,b}(\omega)G(\omega). \tag{9}$$

Then we can say analogously that the above operation of obtaining a quantity $\Delta_{a,b}g(x)$ is the mathematical procedure of obtaining the output $\Delta_{a,b}g(x)$ when the input is given as $g(x)$, and in this case the frequency response of the filter is given by eq. (7). Here, when the impulse response of the filter with its frequency response $K_{a,b}(\omega)$ is denoted by $k_{a,b}(x)$, we have

$$k_{a,b}(x)=\int_{-\infty}^{\infty} K_{a,b}(\omega)e^{i\omega x}d\omega. \tag{10}$$

Therefore, eq. (6) becomes

$$\Delta_{a,b}g(x)=\int_{-\infty}^{\infty} k_{a,b}(x-\xi)g(\xi)d\xi. \tag{11}$$

Above equation indicates that the integral transform with the kernel $k_{a,b}(x)$ of the function $g(x)$ is $\Delta_{a,b}g(x)$.

The operation mentioned above is understood as the limiting case when in eq. (1) the interval S between successive discrete values is approached to 0, and simultaneously α and β are both taken infinitely large, that is,

$$\lim(2\alpha+1)S=2a,\quad \lim(2\beta+1)S=2b.$$

Accordingly, the operation mentioned in this section can be named "the extended running average method". Namely $K_{a,b}(\omega)$ is the characteristic function of such a continuous sampling filter.

In the following the characteristic functions for several values of b will be calculated and their characteristics will be investigated.

The following expressions are obtained easily from eq. (7)′ for several values of n.

(1) For $n=2$

$$K_2^a(\omega)=\frac{\sin a\omega}{a\omega}(1-\cos a\omega). \tag{12}$$

(2) For $n=3$

$$K_3^a(\omega)=\frac{4}{3}\frac{\sin^3 a\omega}{a\omega}. \tag{13}$$

(3) For $n=4$

$$K_4^a(\omega)=\frac{\sin a\omega}{a\omega}(1-\cos a\omega\cdot\cos 2a\omega). \tag{14}$$

In these expressions of the characteristic functions, when the quantities ω_{nM}^a, K_{nM}, C_n and ω^* are introduced, where ω_{nM}^a represents the central angular frequency of the characteristic function $K_n^a(\omega)$ and K_{nM} represents the first maximum value of $K_n^a(\omega)$ and the normalizing constant C_n and the relative frequency ω^* are given by

$$C_n=1/K_{nM} \tag{15}$$

and

$$\omega^*=\omega/\omega_{nM}^a \tag{16}$$

respectively, eq. (12), eq. (13) and eq. (14) give the following expressions.

(1)′ $$K_2^*(\omega^*)=C_2\frac{\sin a\omega_{2M}^a\omega^*}{a\omega_{2M}^a\omega^*}(1-\cos a\omega_{2M}^a\omega^*), \tag{17}$$

where $C_2=1.5$ and $\omega_{2M}^a=1.8182/a$.

(2)′ $$K_3^*(\omega^*)=\frac{4}{3}C_3\frac{\sin^3 a\omega_{3M}^a\omega^*}{a\omega_{3M}^a\omega^*}, \tag{18}$$

where $C_3=1.0838$ and $\omega_{3M}^a=1.3123/a$.

(3)′ $$K_4^*(\omega^*)=C_4\frac{\sin a\omega_{4M}^a\omega^*}{a\omega_{4M}^a\omega^*}(1-\cos a\omega_{4M}^a\omega^*\cos 2a\omega_{4M}^a\omega^*), \tag{19}$$

where $C_4=0.9524$ and $\omega_{4M}^a=1.0526/a$.

These characteristic functions are illustrated in Fig. 2. From Fig. 2 we can understand readily the frequency selectivity characteristics of the present operations as follows: The characteristic of the principal part of $K_2^*(\omega^*)$ is nearly same as that of $K_3^*(\omega^*)$ but a little differ from it of $K_4^*(\omega^*)$. The particular characteristic in the narrow frequency range, as shown in the curve of $K_4^*(\omega^*)$, is not desir-

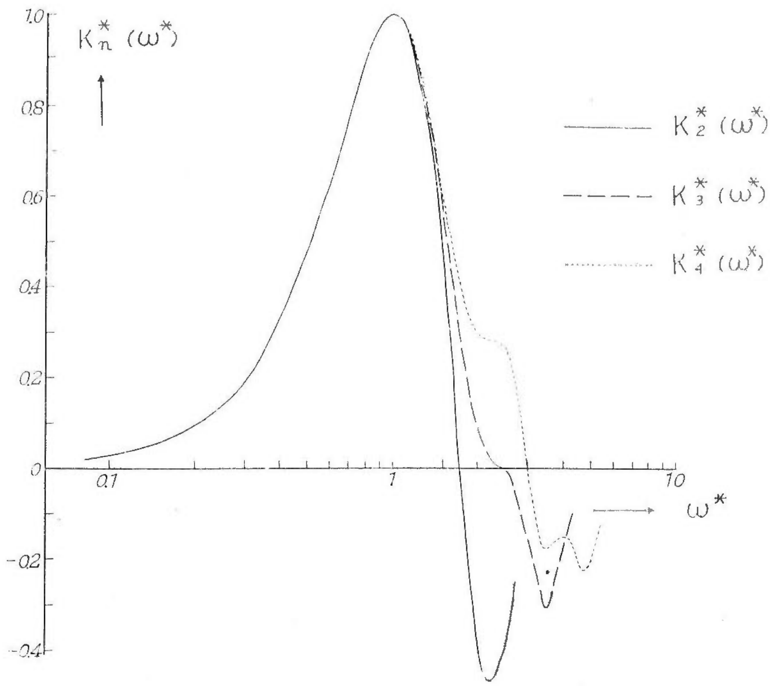

Fig. 2 Filter curves of the extended running average method

able. The presence of this particular characteristic can be easy explained by separating the function $\overset{*}{K_4}(\omega^*)$ into two parts as

$$\overset{*}{K_4}(\omega^*) = C_4\left[\overset{a}{K_2}(\omega) + \overset{2a}{K_2}(\omega)\right]. \tag{20}$$

That is, we can understand that the appearance of the particular characteristic in the curve of $\overset{*}{K_4}(\omega^*)$ is caused by the characteristic of the function $\overset{a}{K_2}(\omega)$ included in $\overset{a}{K_4}(\omega)$. Therefore, we should not adopt the operation with its characteristic of $\overset{a}{K_4}(\omega)$ excepting the special case. As the characteristic of the subordinate part (continued to principal part) of the curves, the characteristic of $\overset{a}{K_4}(\omega)$ is most excellent and the characteristic of $\overset{a}{K_2}(\omega)$ is wrong. Therefore, synthetically speaking, $\overset{a}{K_3}(\omega)$ of these three may be the most excellent characteristic function. Andthe simple expression of $\overset{a}{K_3}(\omega)$ is very convenient when we treat of various problems theoretically.

These characteristic functions of the extended running average method are also expressed as follows respectively:

(1)″ For $b=2a$,

$$\overset{*}{K_2}(\lambda^*) = C_2\frac{\overset{a}{\lambda}_{2M}\lambda^*}{2\pi a}\sin\left(\frac{2\pi a}{\overset{a}{\lambda}_{2M}\lambda^*}\right)\left[1-\cos\left(\frac{2\pi a}{\overset{a}{\lambda}_{2M}\lambda^*}\right)\right] \tag{21}$$

where $\lambda_{2M}^{a}=3.4557a$ and λ^* represents the relative wave length (relative period), i. e., $\lambda^*=\lambda/\lambda_{2M}=1/\omega^*$.

(2)″ For $b=3a$,

$$K_3^*(\lambda^*)=\frac{4}{3}C_3\frac{\lambda_{3M}^{a}\lambda^*}{2\pi a}\sin^3\left(\frac{2\pi a}{\lambda_{3M}^{a}\lambda^*}\right) \tag{22}$$

where $\lambda_{3M}^{a}=4.7879a$.

(3)″ For $b=4a$,

$$K_4^*(\lambda^*)=C_4\frac{\lambda_{4M}^{a}\lambda^*}{2\pi a}\sin\left(\frac{2\pi a}{\lambda_{4M}^{a}\lambda^*}\right)\left[1-\cos\left(\frac{2\pi a}{\lambda_{4M}^{a}\lambda^*}\right)\cos\left(\frac{4\pi a}{\lambda_{4M}^{a}\lambda^*}\right)\right], \tag{23}$$

where $\lambda_{4M}^{a}=5.9692a$.

So these functions will be called as "the normalized selection coefficient". In Fig. 3, these normalized selection coefficients are shown.

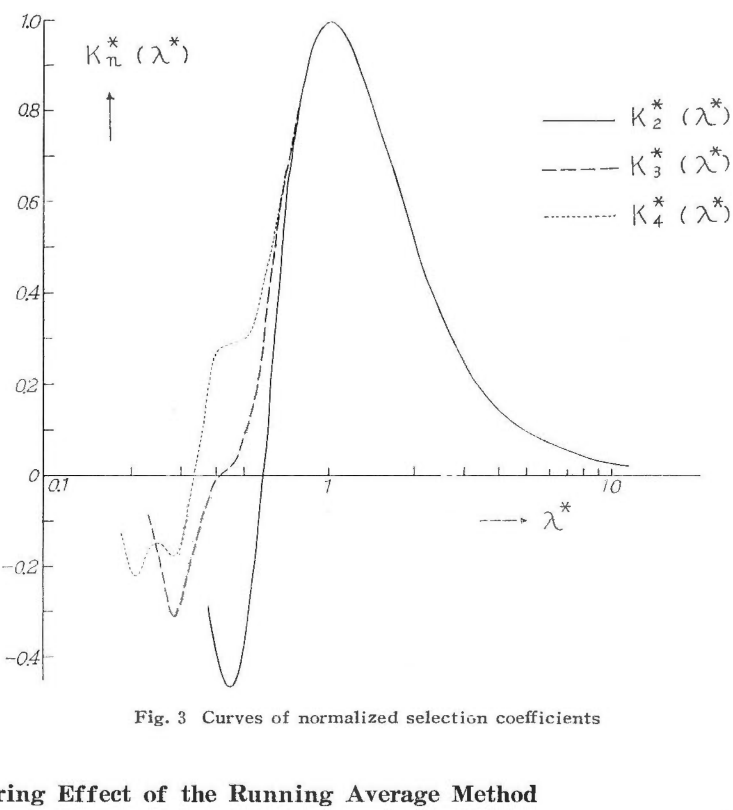

Fig. 3 Curves of normalized selection coefficients

II. 3 Filtering Effect of the Running Average Method

In many cases physical values observed are given as values at discrete measurement points. Then in the following, the case in which the running average method is applied to series of the values observed at consecutive equally spaced points will be considered. In this case the detected quantities are expressed by

$$\Delta_{\alpha,\beta}g(x)=\int_{-\infty}^{\infty}K_{\alpha,\beta}(\omega)G(\omega)e^{i\omega x}d\omega \tag{24}$$

or

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K_{\alpha,\beta}(\omega)g(\xi)e^{i\omega(x-\xi)}d\xi\ d\omega, \tag{25}$$

where

$$K_{\alpha,\beta}(\omega)=\frac{\sin\frac{(2\alpha+1)}{2}S\omega}{(2\alpha+1)\sin\frac{S\omega}{2}}-\frac{\sin\frac{(2\beta+1)}{2}S\omega}{(2\beta+1)\sin\frac{S\omega}{2}} \tag{26}$$

The function given by eq. (26) is the characteristic function of the present selective sampling operation, namely, of the running average method defined by eq. (1).

We must pay attention to that we have not any information of physical quantity $g(x)$ in ranges excepting the discrete measurement points, accordingly, the true function of $g(x)$ is not known. Therefore, we must assume adequate function $\tilde{\tilde{g}}(x)$ which may approximate to the true function instead of $g(x)$ in eq. (26) by using observed values.

Then the continuous function expressed by the following equation is considered.

$$\tilde{\tilde{g}}(x)=\int_{-\Omega}^{\Omega}\tilde{\tilde{G}}(\omega)e^{i\omega x}d\omega. \tag{27}$$

And we assume that this function has the values g_n at the measurement points P_n, moreover, we choose the value of Ω as $\Omega=\pi/S$. Then we have

$$g_n=g(nS)=\int_{-\pi/S}^{\pi/S}\tilde{\tilde{G}}(\omega)e^{i\omega nS}d\omega. \tag{28}$$

Therefore, the function $\tilde{\tilde{g}}(x)$ expressed by eq. (27) is a continuous function which approximate to true function and coincide to observed values at each measurement point, and not include a variation of wave length below λ_S ($\lambda_S=2S$). Tomoda and Sensyu[23] indicated that this approximate function is expressed by

$$\tilde{\tilde{g}}(x)=\Sigma g(nS)\frac{\sin\frac{\pi}{S}(x-nS)}{\frac{\pi}{S}(x-nS)} \tag{29}$$

and they also indicated that the Fourier transform of $g(x)$ is expressed by

$$\tilde{\tilde{G}}(\omega)=\frac{S}{2\pi}\Sigma\, g(nS)e^{-i\omega nS}. \tag{30}$$

So we can have the following equation instead of eq. (24) and eq. (25) respectively:

$$\Delta_{\alpha,\beta}\tilde{\tilde{g}}(x)=\int_{-\infty}^{\infty}K_{\alpha,\beta}(\omega)\tilde{\tilde{G}}(\omega)e^{i\omega x}d\omega \tag{24'}$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K_{\alpha,\beta}(\omega)\tilde{\tilde{g}}(\xi)e^{i\omega(x-\xi)}d\xi\ d\omega. \tag{25'}$$

Then the spectrum of $\Delta_{\alpha,\beta}\tilde{\tilde{g}}(x)$ is given by

$$\Delta_{\alpha,\beta}\tilde{\tilde{G}}(\omega)=K_{\alpha,\beta}(\omega)\tilde{\tilde{G}}(\omega). \tag{31}$$

Next we will investigate the characteristics of various normalized selection coefficients given by general form of

$$K^*_{\alpha,\beta}(\lambda)=C_{\alpha,\beta}\left[\frac{\sin(2\alpha+1)\pi S/\lambda}{(2\alpha+1)\sin\pi S/\lambda}-\frac{\sin(2\beta+1)\pi S/\lambda}{(2\beta+1)\sin\pi S/\lambda}\right], \tag{32}$$

where $C_{\alpha,\beta}$ denotes the normalizing constant.

In Fig. 4 these various normalized selection coefficients are shown. From Fig. 4 it is evident that the most excellent selection coefficient of these is $K^*_{2,7}(\lambda)$, but if we consider the detection for a variation of smaller scale than the former, the

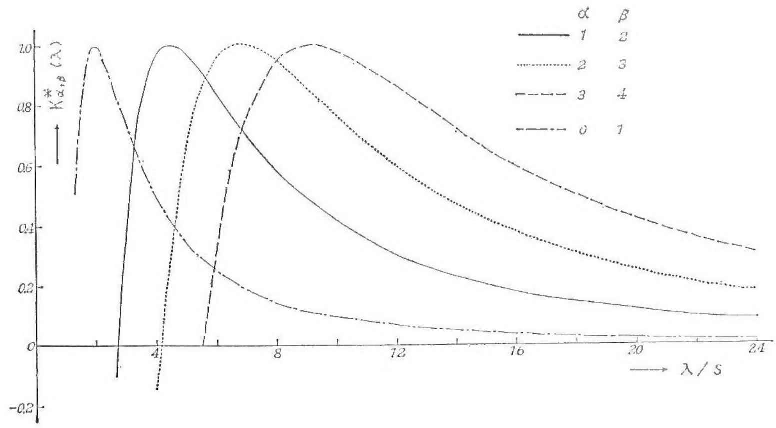

Fig. 4-a Normalized selection coefficients of various detections of the running average method (principal part)

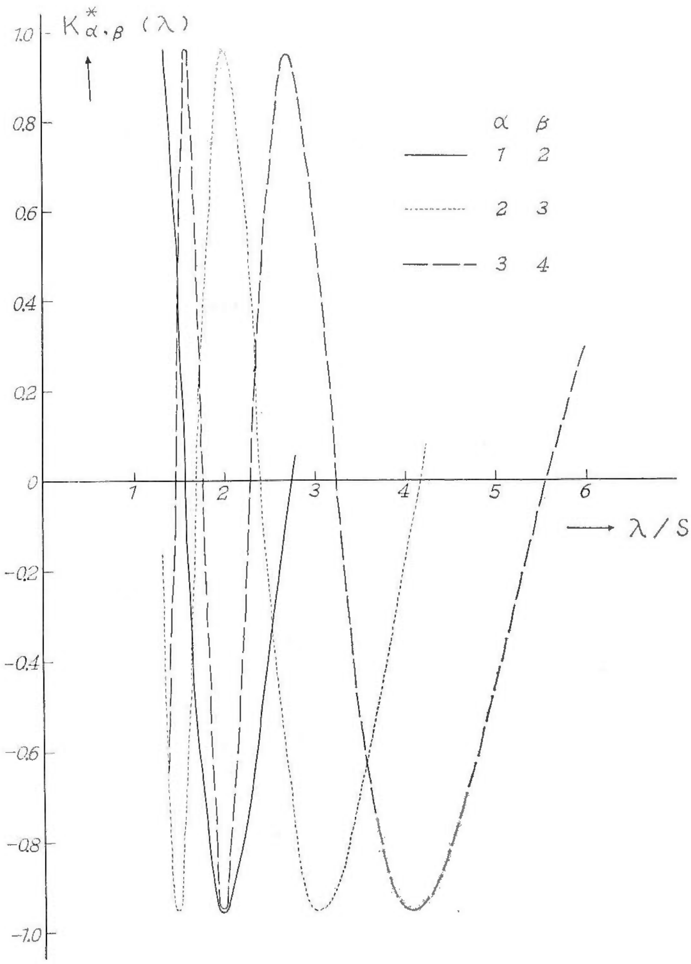

Fig. 4-b ditto (subordinate part)

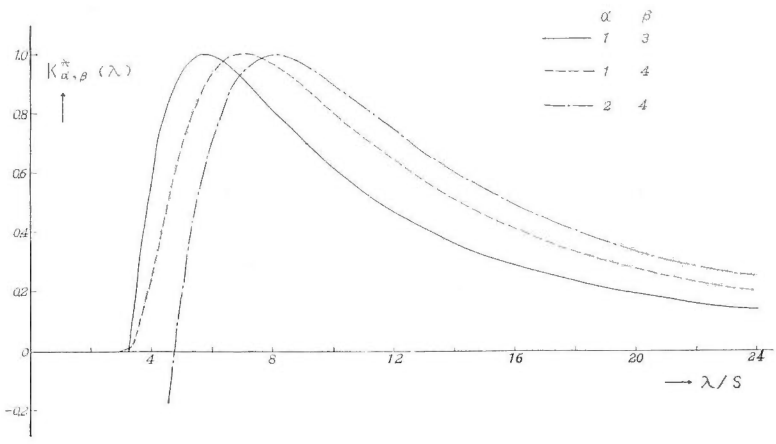

Fig. 4-c ditto (principal part)

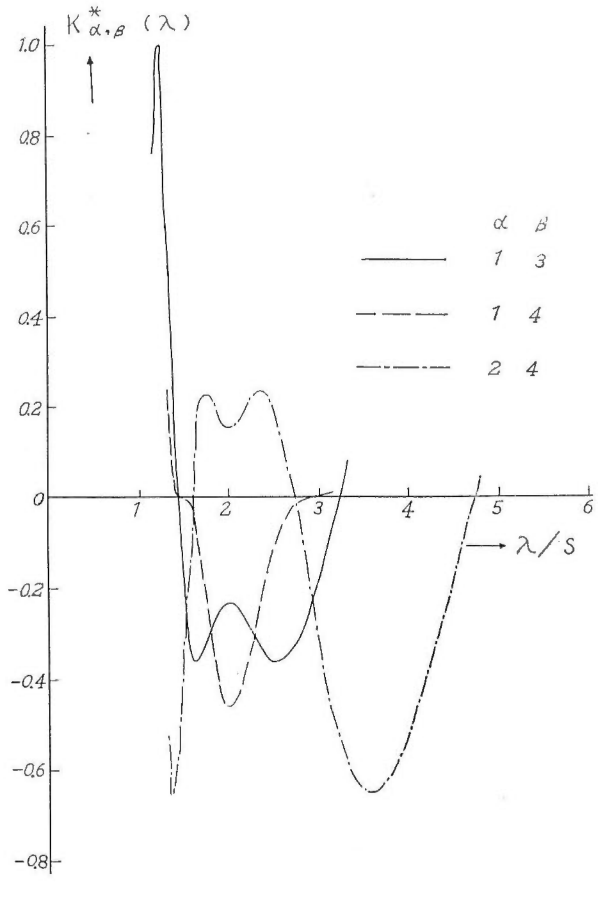

Fig. 4-d ditto (subordinate part)

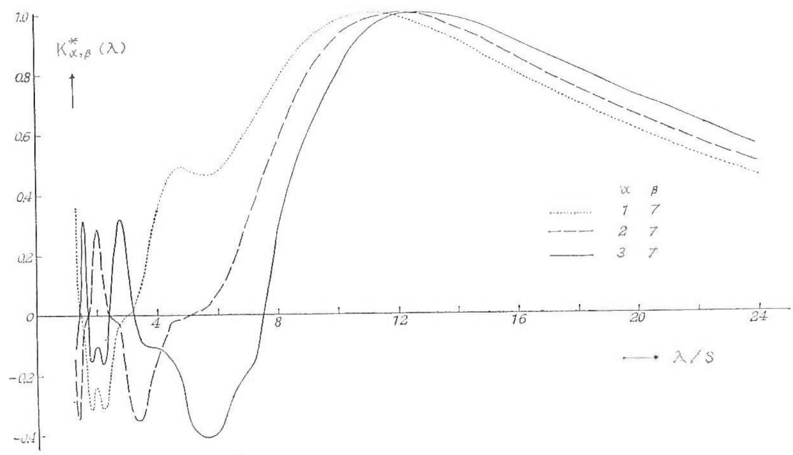

Fig. 4-e ditto

characteristic of $K^*_{1,4}$ (λ) is most excellent. And the characteristics of $K^*_{1,3}$ (λ) and $K^*_{3,7}$ (λ) are somewhat inferior to $K^*_{1,4}$ (λ) and $K^*_{2,7}$ (λ) respectively. However, it is supposed that in the actual use the differences of these characteristics are negligible. In the previous paper the writer adopted the detections with those characteristics $K_{1,3}$ (λ) and $K_{3,7}$ (λ) as the method of analysis in the gravity prospecting, and he named "the normal detection" and "the bi-structural detection" for each detection respectively.

In Fig. 5 these excellent selection coefficients are shown in normalized expres-

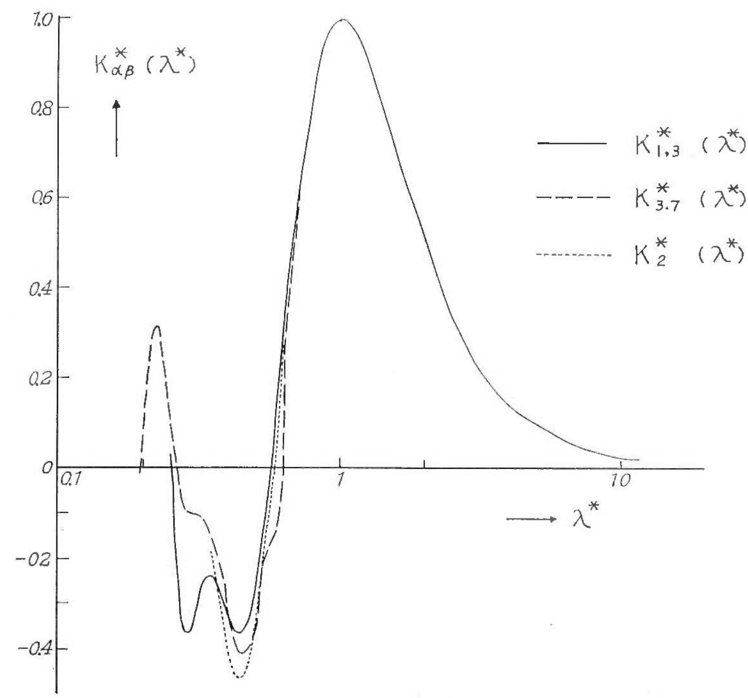

Fig. 5-a Curves of normalized selection coefficients

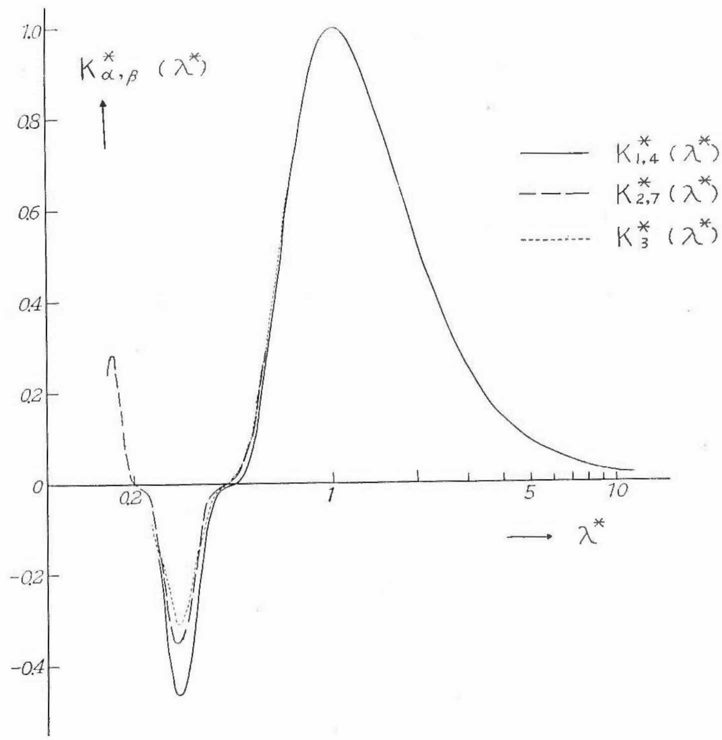

Fig. 5-b Curves of normalized selection coefficients

sion and by the use of the relative wave length. The general form of these coefficients is given by

$$K^*_{\alpha,\beta}(\lambda^*)=C_{\alpha,\beta}\left\{\frac{\sin\frac{(2\alpha+1)\pi S}{\lambda_M}\frac{1}{\lambda^*}}{(2\alpha+1)\sin\frac{\pi S}{\lambda_M}\frac{1}{\lambda^*}}-\frac{\sin\frac{(2\beta+1)\pi S}{\lambda_M}\frac{1}{\lambda^*}}{(2\beta+1)\sin\frac{\pi S}{\lambda_M}\frac{1}{\lambda^*}}\right\}, \tag{33}$$

where λ_M represents the central wave length. From Fig. 5 it is evident that the characteristic of $K^*_{1,3}(\lambda^*)$ is nearly same to the characteristic of $K^*_{3,7}(\lambda^*)$ and similarly the characteristic of $K^*_{1,4}(\lambda^*)$ is nearly same to that of $K^*_{2,7}(\lambda^*)$, and moreover, all the principal parts of these are nearly coincident each other. The latter fact is also present, as pointed out in the previous paper, even for other various values of α and β. In addition, the curves of dotted line in Fig. 5 (a) and (b) are the curves of $K^*_2(\lambda^*)$ and $K^*_3(\lambda^*)$ respectively. Then we can understand that the difference between any selective sampling and a corresponding continuous sampling to it may be little so far as we do not consider variations of small scale. That is, as shown in Table 1, for the normal detection the ratio $(2\beta+1)/(2\alpha+1)$ is equal to 2.333 and for the bi-structural detection this ratio is equal to 2.143. Therefore, it is supposed that these characteristics may approximate to the characteristic of $K^*_2(\lambda^*)$. Fig. 5 (a) shows that in the practice the degree of this approximation is as higher as we can regard these as identical. For the case of Fig. 5 (b) we can also say similarly.

Thus it becomes clear that the characteristic functions of the normal detection and bi-structural detection can be approximated by the use of the characteristic function $K_2^{a}(\omega)$, and that the characteristic functions $K_{1,4}(\omega)$ and $K_{2,7}(\omega)$ can be approximated by the use of $K_3^{a}(\omega)$. For the purpose of performing the approximations above mentioned, we must take the values of a in each characteristic function $K_2^{a}(\omega)$ and $K_3^{a}(\omega)$ as follows:

That is, when the central wave length of the selection coefficient $K_{\alpha,\beta}(\lambda)$ is denoted by the symbol $\lambda_{\alpha,\beta M}$, it is necessary for upper purpose that the following relations are present.

$$\lambda_{\alpha,\beta M}=\lambda^{a}_{nM} \qquad (n=2 \text{ and } 3).$$

Then when denote the value of a satisfying above relation by the symbol $a^{n}_{\alpha,\beta}$, we have

$$a^{2}_{\alpha,\beta}=0.2894\,\lambda_{\alpha,\beta M} \qquad \text{for } n=2 \tag{34}$$

and

$$a^{3}_{\alpha,\beta}=0.2089\,\lambda_{\alpha,\beta M} \qquad \text{for } n=3. \tag{34}'$$

In Table 1 the central wave length $\lambda_{\alpha,\beta M}$, the ratio $(2\beta+1)/(2\alpha+1)$ (correspond to b/a in continuous sampling) and $2\lambda_{\alpha,\beta M}/(2\alpha+1)S$ (correspond to λ^{a}_{nM}/a in continuous sampling) are tabulated. And in Table 2 $a^{n}_{\alpha,\beta}$ and $b^{n}_{\alpha,\beta}=na^{n}_{\alpha,\beta}$ are tabulated.

Table 1 Values of the central wave length of the various detections

α	β	$\frac{2\beta+1}{2\alpha+1}$	$\lambda_{\alpha,\beta M}$	$\frac{2\lambda_{\alpha,\beta M}}{(2\alpha+1)S}$
1	3	2.333	5.7 S	3.80
1	4	3	6.9 S	4.60
2	7	3	12.0 S	4.80
3	7	2.143	12.4 S	3.54

Table 2 Inner parts of brackets indicate the values which were calculated formally by respective relations of eq. (34) and eq. (34)′

α	β	$a^{3}_{\alpha,\beta}$	$a^{2}_{\alpha,\beta}$	$b^{3}_{\alpha,\beta}$	$b^{2}_{\alpha,\beta}$
1	3	(1.188 S)	1.650 S	(3.564 S)	3.299 S
1	4	1.438 S	(1.997 S)	4.314 S	(3.993 S)
2	7	2.500 S	(3.473 S)	7.500 S	(6.945 S)
3	7	(2.583 S)	3.588 S	(7.749 S)	7.177 S

By using these values of $a^{n}_{\alpha,\beta}$ and $b^{n}_{\alpha,\beta}$ we can perform nearly the same detection as the detection of the selection coefficient $K_{\alpha,\beta}(\lambda)$. Though, in this case we should take the continuous function $g(x)$ which does not include a variation of a wave length below the wave length $\lambda_S=2S$ instead of the true function $g(x)$. In the following, this matter will be proved analytically.

Now we introduce the new function

$$K_{\alpha,\beta}(\omega)=C^{n}_{\alpha,\beta}\;K^{a}_{n}\;(\omega), \tag{35}$$

where

$$C^{n}_{\alpha,\beta}=C_n/C_{\alpha,\beta}. \tag{36}$$

Then by using this characteristic function $K_{\alpha,\beta}\,(\omega)$ instead of $K_{\alpha,\beta}\,(\omega)$ in the integrand of eq. (25)′, we have

$$\begin{aligned} \varDelta_{\alpha,\beta}\,\tilde{\tilde{g}}(x) &\approx \int_{-\infty}^{\infty} K_{\alpha,\beta}(\omega)\tilde{\tilde{G}}(\omega)\;e^{i\omega x}d\omega \\ &= \frac{S}{2\pi}\sum g(nS)\int_{-\infty}^{\infty} K_{\alpha,\beta}(\omega)\;e^{i\omega(x-nS)}d\omega. \end{aligned} \tag{37}$$

By the use of eq. (10), the right hand of above equation becomes

$$\varDelta_{\alpha,\beta}\,\tilde{\tilde{g}}(x)\approx S\sum C_{\alpha,\beta}\,k_{a\alpha,\beta}\,(x-nS)\;g(nS). \tag{38}$$

In Table 3 (in the 2nd and 3rd column), the kinds of selective sampling equivalent to the continuous sampling that the right hand of eq. (37) indicates are compared with those of the selective sampling corresponding to each case.

Here, for reference the case in which $\varDelta_{\alpha,\beta}\,\tilde{\tilde{g}}(x)$ is approximated by the use of the detection of its characteristic function $\overset{a\alpha}{K_n}\,(\omega)$ will be investigated, where $a_\alpha=(2\alpha+1)S/2$. In this case, we have

$$\varDelta_{\alpha,\beta}\,\tilde{\tilde{g}}(x)\approx\int_{-\infty}^{\infty} K_{a_\alpha}(\omega)\;\tilde{\tilde{G}}(\omega)\;e^{i\omega x}d\omega, \tag{39}$$

where

$$K_{a_\alpha}(\omega)=C_{\alpha,\beta}\,\overset{a\alpha}{K}_{n}(\omega). \tag{40}$$

By similar treatment as before, we have

$$\varDelta_{\alpha,\beta}\,\tilde{\tilde{g}}(x)\approx S\sum C_{\alpha,\beta}\,k_{a_\alpha}(x-nS)\;g(nS). \tag{41}$$

Table 3 Equivalent selective sampling

α	β	$\overset{3}{a}_{\alpha,\beta}$ α	β	$\overset{2}{a}_{\alpha,\beta}$ α	β	$\overset{3}{a}_{\alpha}$ α	β	$\overset{2}{a}_{\alpha}$ α	β	$\overset{3}{\lambda}_M$	$\overset{2}{\lambda}_M$
1	3	1	3	1	3	1	4	1	3	7.18 S	5.18 S
1	4	1	4	1	3	1	4	1	3	7.18 S	5.18 S
2	7	2	7	3	6	2	7	2	5	11.97 S	8.64 S
3	7	2	7	3	7	3	10	3	7	16.76 S	12.10 S

In the other part of Table 3 the results of this case are tabulated. Moreover, $\overset{n}{\lambda}_M$ in each column means the central wave length of the corresponding detections.

II. 4 Filtering Effect of the Running Average Method for Periodic Function

It is not general case in which a physical quantity has periodicity. Although, even in the case in which $g\,(x)$ possesses a continuous spectrum, in many cases we often assume that $g\,(x)$ possesses a line spectrum for reason of its theoretical convenience. The characteristics of the procedure of the running average method in this case is given by the expression of the same form as the characteristic function obtained in previous section excepting the difference of line spectrum and continuous spectrum. That is, in the presence the following equation corresponding to eq. (25) in the previous section is obtained.

$$\Delta_{\alpha,\beta}\, g(x) = \sum_{n=0}^{\infty} \frac{\varepsilon_n}{L} \int_0^L K_{\alpha,\beta}(\omega_n)\, g(\xi) \cos \omega_n (x-\xi)\, d\xi, \tag{42}$$

where

$$\varepsilon_0 = 1, \qquad \varepsilon_1 = \varepsilon_2 = \cdots\cdots = 2$$

and

$$\omega_n = 2\,\pi n/L,$$

and $K_{\alpha,\beta}\,(\omega)$ is the characteristic function given by the following equation

$$K_{\alpha,\beta}(\omega_n) = \frac{\sin\frac{(2\alpha+1)\omega_n S}{2}}{(2\alpha+1)\sin\frac{\omega_n S}{2}} - \frac{\sin\frac{(2\beta+1)\omega_n S}{2}}{(2\beta+1)\sin\frac{\omega_n S}{2}}. \tag{43}$$

This is the expression of the same form as eq. (26).

III. Application to the Two Dimensional Problems of the Running Average Method

III. 1 Case of the Selective Sampling

In this section, the case in which the observed values are given intermittently, as shown in Fig. 6, is considered. Actual distribution of observed points is uni-

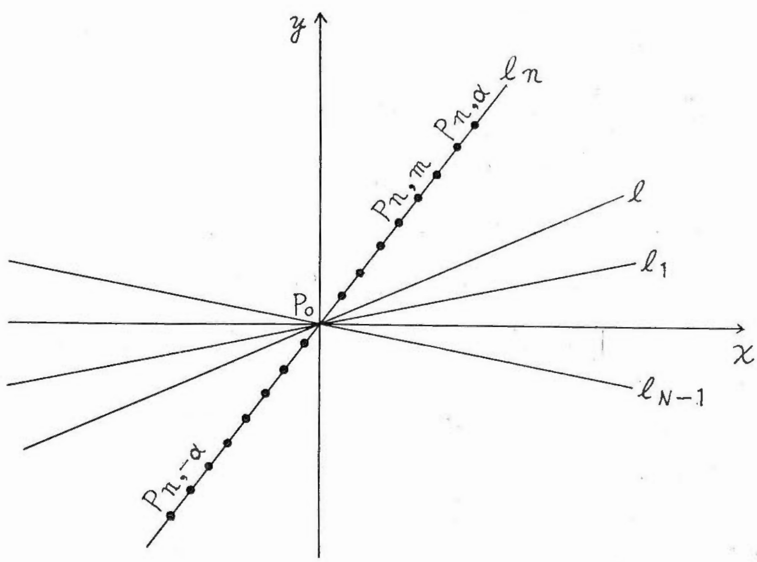

Fig. 6 Illustration of selective sampling of the two dimensional case

form, but in Fig. 6, unnecessary points are neglected for simplicity. At first the mean of the measured values at $(2\alpha+1)$ points with the center point $P_0\,(x_i, y_j)$ on each measurement line l_n $(n=1, 2, 3\cdots\cdots, \mathrm{N})$ is computed, and then next the sum S_α of these means is also computed. When the average value of the sum S_α for the measurement lines is denoted by $\bar{S}_\alpha^N$, this average value is given by

$$\bar{S}_\alpha^N = \frac{1}{(2\alpha+1)N} \sum_{n=1}^{N} \sum_{m=-\alpha}^{\alpha} g_{n,m}, \tag{44}$$

where $g_{n,m}$ represents the observed value at the point $P_{n,m}$ on the line l_n, and $g_{n,o}$

is equal to the value, g_o, at the center point P_o, i.e.,

$$g_{1,o}=g_{2,o}=\cdots\cdots g_{N,o}=g_o.$$

Eq. (44) is also expressed by the simpler form of

$$\bar{S}_\alpha^N=\frac{1}{(2\alpha+1)}\Big\{g_o+2\sum_{m=1}^{\alpha}\bar{g}_m^N\Big\}, \tag{45}$$

where $\bar{g}_m^N$ represents the mean of the values on the circle of its radius $r=mS$. In the following, the average value S will be calculated analytically.

The physical quantity $g\ (x, y)$ is expressed by the Fourier transform of

$$g\ (x, y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}G(\omega_x, \omega_y)\ e^{i(\omega_x x+\omega_y y)}\,d\omega_x\, d\omega_y. \tag{46}$$

And when we put

$$x=x_i+r\cos\nu\theta,\qquad y=y_j+r\sin\nu\theta$$

and

$$g\ (x, y)=g\ (r, \nu\theta),$$

eq. (46) becomes

$$g(r, \nu\theta)=\int_0^{2\pi}\int_0^{\infty}G\ (\omega, \phi)\ e^{i\omega(x_i\cos\phi+y_j\sin\phi)}e^{ir\omega\cos(\nu\theta-\phi)}\times\omega d\omega d\phi, \tag{47}$$

where

$$\omega^2=\omega_x{}^2+\omega_y{}^2,\qquad \phi=\tan^{-1}\frac{\omega_y}{\omega_x}$$

and

$$G\ (\omega, \phi)=G\ (\omega_x, \omega_y).$$

Therefore,

$$\bar{g}_r^N=\frac{1}{2N}\int_0^{2\pi}\int_0^{\infty}\omega G\ (\omega, \phi)\ e^{i\omega(x_i\cos\phi+y_j\sin\phi)} \times\sum_{\nu=1}^{2N}e^{ir\omega\cos(\nu\theta-\phi)}\,d\omega\, d\phi. \tag{48}$$

By using this equation, the average value $\bar{S}_\alpha^N$ is given as

$$\bar{S}_\alpha^N=\frac{1}{(2\alpha+1)}\int_0^{2\pi}\int_0^{\infty}\omega G\ (\omega, \phi)\ e^{i\omega(x_i\cos\phi+y_j\sin\phi)} \times\Big[1+2\{J_N(S\omega)+J_N(2S\omega)+\cdots+J_N(\alpha S\omega)\}\Big]\,d\omega\, d\phi, \tag{49}$$

where

$$J_N(r\omega)=\frac{1}{2N}\sum_{\nu=1}^{2N}e^{ir\omega\cos(\nu\theta-\phi)}. \tag{50}$$

Now, if the quantity

$$\Delta_{\alpha,\beta}\,g\ (x_i, y_j)=\bar{S}_\alpha^N-\bar{S}_\beta^N$$

is considered, this quantity is given by using eq. (49) as

$$\Delta_{\alpha,\beta}\,g\ (x_i, y_j)=\int_0^{2\pi}\int_0^{\infty}\omega G\ (\omega, \phi)\ K_{\alpha,\beta}^{(N)}(\omega) \times e^{i\omega(x_i\cos\phi+y_j\sin\phi)}\,d\omega\, d\phi, \tag{51}$$

where

$$K_{\alpha,\beta}^{(N)}(\omega)=\frac{2(\beta-\alpha)}{(2\alpha+1)(2\beta+1)}+\frac{4(\beta-\alpha)}{(2\alpha+1)(2\beta+1)}\sum_{\kappa=1}^{\alpha}J_N(kS\omega)-\frac{2}{(2\beta+1)}\sum_{k=\alpha+1}^{\beta}J_N(kS\omega) \tag{52}$$

and this is the characteristic function of the running average method in the two dimensional case.

In the practice, N is taken as small number. In such a case, $J_N\ (r\omega)$ can be expressed by simple form as follows:

For $N=2$

$$J_2(r\omega)=\frac{1}{2}\left\{\cos(r\omega_x)+\cos(r\omega_y)\right\}. \tag{53}$$

For $N=3$

$$J_3(r\omega)=\frac{1}{3}\left\{\cos(r\omega_x)+2\cos\left(\frac{r}{2}\omega_x\right)\cos\left(\frac{\sqrt{3}}{2}r\omega_y\right)\right\}. \tag{54}$$

For $N=4$

$$J_4(r\omega)=\frac{1}{4}\left\{\cos(r\omega_x)+2\cos\left(\frac{r}{\sqrt{2}}\omega_x\right)\cos\left(\frac{r}{\sqrt{2}}\omega_y\right)+\cos(r\omega_y)\right\}. \tag{55}$$

Next, the ideal case will be considered, in which the value of N is taken as infinitely large, then eq. (50) gives

$$\lim_{N\to\infty} J_N(r\omega)=\frac{1}{2\pi}\int_0^{2\pi} e^{ir\omega\cos(\theta-\phi)}\,d\theta=J_0(r\omega),$$

where $J_0(r\omega)$ represent the Bessel function of 0 th order. In this case, the idealized characteristic function $K^{\infty}_{\alpha,\beta}(\omega)$ is given by

$$K^{\infty}_{\alpha,\beta}(\omega)=\frac{2(\beta-\alpha)}{(2\alpha+1)(2\beta+1)}+\frac{4(\beta-\alpha)}{(2\alpha+1)(2\beta+1)}\sum_{\kappa=1}^{\alpha} J_0(kS\omega)-\frac{2}{(2\beta+1)}\sum_{\kappa=\alpha+1}^{\beta} J_0(kS\omega). \tag{56}$$

For example, for the normal detection in the two dimensional case, it becomes

$$K^{\infty}_{1,3}(\omega)=\frac{2}{21}\left\{2+4J_0(S\omega)-3J_0(2S\omega)-3J_0(3S\omega)\right\}. \tag{57}$$

In Fig. 7, this characteristic function is indicated in normalized and relative wave length expression. The curve of dashed line in Fig. 7 is the filter curve of the normal detection of the one dimensional case.

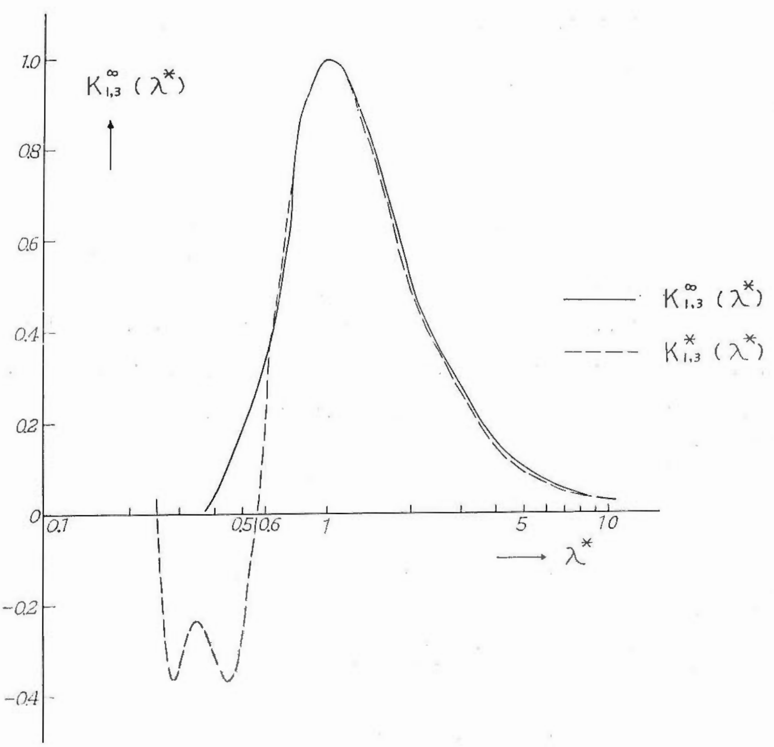

Fig. 7 Filtering curve of the normal detection in the two dimensional case (central wave length λ_M=4.6496 S)

III. 2 Case of the Continuous Sampling

Eq. (56) is the equation of the characteristic function of the case in which the continuous sampling procedure is performed for the mean on a circle. In this section, the case in which the continuous sampling procedure is also applied to the radius direction will be considered.

When the characteristic function in this case is denoted by $K^{\infty}_{ab}(\omega)$, this characteristic function is expressed by

$$K^{\infty}_{ab}(\omega)=\frac{1}{2a}\int_0^a J_0(r\omega)\,dr-\frac{1}{2b}\int_0^b J_0(r\omega)\,dr. \tag{58}$$

Here, it must be attended to that the anomalous quantity obtained by the detection with its characteristic $K^{\infty}_{ab}(\omega)$ is represented by

$$\Delta_{ab}\,g(x,y)=\frac{1}{2\pi a}\int_0^{2\pi}\int_0^a g(r,\theta)\,dr\,d\theta-\frac{1}{2\pi b}\int_0^{2\pi}\int_0^b g(r,\theta)\,dr\,d\theta \tag{59}$$

but is not the difference of two average values of usual meaning for different circles, namely, is not the quantity

$$\Delta^*_{ab}\,g(x,y)=\frac{1}{\pi a^2}\int_0^{2\pi}\int_0^a g(r,\theta)\,r\,dr\,d\theta-\frac{1}{\pi b^2}\int_0^{2\pi}\int_0^b g(r,\theta)\,r\,dr\,d\theta. \tag{60}$$

In the following, the characteristic function of the detection defined by eq. (60) will be investigated.

The integral

$$I_a=\int_0^{2\pi}\int_0^a g(r,\theta)\,r\,dr\,d\theta$$

is given by the Fourier transform expression of

$$I_a=2\pi a^2\int_0^{2\pi}\int_0^{\infty}\frac{J_1(a\omega)}{a\omega}\,G(\omega,\phi)\,e^{i\omega(x\cos\phi+y\sin\phi)}\,d\omega\,d\phi.$$

Therefore, eq. (60) becomes

$$\Delta^*_{ab}\,g(x,y)=\int_0^{2\pi}\int_0^{\infty}K^{\mathrm{II}}_n(\omega)\,G(\omega,\phi)\,e^{i\omega(x\cos\phi+y\sin\phi)}\,d\omega\,d\phi, \tag{61}$$

where

$$K^{\mathrm{II}}_n(\omega)=\frac{2}{na\omega}\Big[nJ_1(a\omega)-J_1(na\omega)\Big],\qquad n=\frac{b}{a} \tag{62}$$

and this is the characteristic function in this case.

In Fig. 8, the normalized selection coefficients for $b=2a$ and for $b=3a$, i. e., $K^{\mathrm{II}*}_2(\lambda^*)$ and $K^{\mathrm{II}*}_3(\lambda^*)$ respectively, are shown by using the relative wave length λ^*. And the expressions of these coefficients are respectively

(1) for $b=2a$,

$$K^{\mathrm{II}*}_2(\lambda^*)\doteqdot\frac{1}{2}C^{\mathrm{II}}_2\lambda^*\Big[2J_1\Big(\frac{2}{\lambda^*}\Big)-J_1\Big(\frac{4}{\lambda^*}\Big)\Big], \tag{63}$$

where

$$C^{\mathrm{II}}_2=1.6402 \quad\text{and}\quad \lambda_{2M}\doteqdot\pi a,$$

(2) for $b=3a$,

$$K^{\mathrm{II}*}_3(\lambda^*)\doteqdot\frac{4}{9}C^{\mathrm{II}}_3\lambda^*\Big[3J_1\Big(\frac{3}{2\lambda^*}\Big)-J_1\Big(\frac{9}{2\lambda^*}\Big)\Big], \tag{64}$$

where

$$C^{\mathrm{II}}_3=1.1811 \quad\text{and}\quad \lambda_{3M}\doteqdot\frac{4}{3}\pi a.$$

The characteristics of these two are nearly equal each other for the values of $\lambda^*>0.7$, but for the values of $\lambda^*<0.7$ the difference of the characteristics between

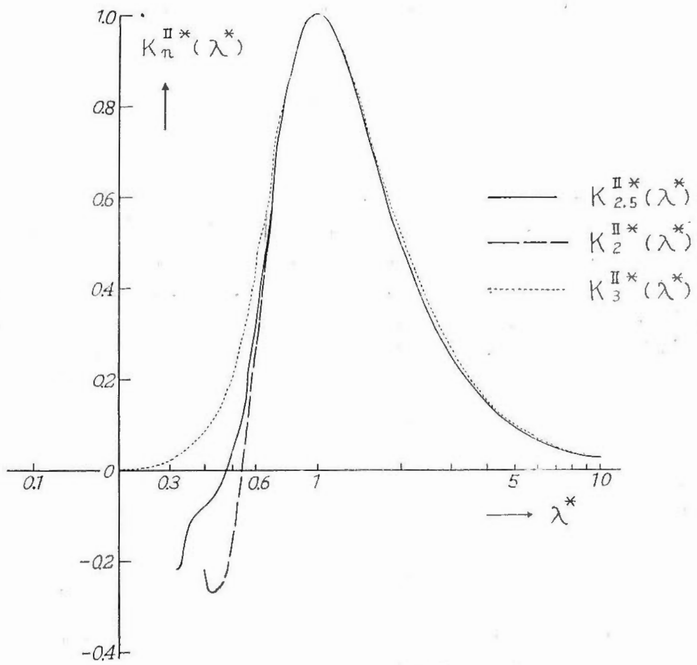

Fig. 8 Curves of $K_n^{II*}(\lambda^*)$

these two is large as shown in Fig. 8. On the character of high cut it is concluded that the intermediate characteristic of these two is desirable. The curve of solid line in Fig. 8 is the filter curve of which function is

$$K_{2\cdot5}^{II*}(\lambda^*) \doteqdot \frac{7C_{2\cdot5}^{II}}{30}\lambda^*\left[5J_1\left(\frac{12}{7\lambda^*}\right)-2J_1\left(\frac{30}{7\lambda^*}\right)\right], \tag{65}$$

where

$$C_{2\cdot5}^{II}=1.3228 \quad \text{and} \quad \lambda_{2\cdot5M} \doteqdot \frac{7}{6}\pi a.$$

The characteristic of this function is very similar to that of $K_3^*(\lambda^*)$ in the one dimensional case excepting the range of small λ^*, but the characteristic of the former in the range of small λ^* is more excellent than that of the latter.

From the considerations mentioned above, it is concluded that the adoptation of the sampling procedure with its characteristic $K_{2\cdot5}^{II}(\omega)$ is desired for two dimensional problems if it is possible. However, practically it is not necessarily required the best treatment theoretically, but rather in many cases a simple treatment is required even if it were more or less rough treatment. An analysis of gravity is this case, then in the analysis of the gravity prospecting the running average method of $N=2$ is always used. This calculation is very simple, and it seems that the characteristic of this detection will not become very worse than $K_{2\cdot5}^{II}(\omega)$ in spite of its simplicity presuming from the characteristic of eq. (57).

When we consider theoretically gravity anomalies detected by the use of the running average method, the characteristic function $K_2^{II}(\omega)$ can be adopted approximately as the characteristic function in two dimensional case instead of the true characteristic function, as in one dimensional case, by reason of its simple analytical expression.

IV. Meanings of Quantities detected by the Running Average Method

IV. 1 Superficial Meaning

From the considerations of the characteristic functions of the running average method, it is concluded that this method has the filtering effect with the character of a band pass filter and so the anomalous quantities obtained by this method are not including noises (errors and variations of small scale) and the variations of large scale. From the definition of the running average method the same conclusion is also obtained directly as follows:

From the definition

$$\Delta_{\alpha,\beta}\, g(x)=\bar{g}^{\alpha}(x)-\bar{g}^{\beta}(x).$$

Therefore,

$$\bar{g}^{\alpha}(x)=\Delta_{\alpha,\beta}\, g(x)+\bar{g}^{\beta}(x).$$

Subtracting each side from $g(x)$, we have

$$g(x)-\bar{g}^{\alpha}(x)=g(x)-\{\Delta_{\alpha,\beta}\, g(x)+\bar{g}^{\beta}(x)\}.$$

From the definition the left side of above equation becomes

$$g(x)-\bar{g}^{\alpha}(x)=\Delta_{0,\alpha}\, g(x).$$

This quantity is obtained by applying the detection with the characteristic function $K_{0,\alpha}(\omega)$, and include errors and variations of small scale, i.e., this is so-called noise. Therefore,

$$\Delta_{\alpha,\beta}\, g(x)=g(x)-\{\Delta_{0,\alpha}\, g(x)+\bar{g}^{\beta}(x)\}. \tag{66}$$

Eq. (66) means that the quantity $\Delta_{\alpha,\beta}\, g(x)$ is the quantity which is obtained by removing the tendency of large scale and noises from the observed value $g(x)$.

IV. 2 Meaning as Indication

Bouguer anomaly $g(x)$ is caused by the subterranean density distribution, in other words, the gravity distribution on the earth surface can be calculated by performing the integral transformation to all the subterranean density distribution $\rho(x,z)$. Similarly, also when $g(x)$ is an arbitrary physical quantity, it is considered that this quantity is produced as all effects for variable z of quantities which are obtained by any integral transformation from the physical quantity $\rho(x,z)$ of the same kind or the different kind. In this case what is the physical quantity corresponding to anomalous quantity $\Delta g(x)$ obtained by applying the running average method?

Now let the integral transformation in the presence be linear and let the kernel of this integral transformation be

$$l(x,\xi;z)=l(x-\xi,z).$$

So the following general expression is obtained.

$$g(x)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l(x-\xi,z)\,\rho(\xi,z)\,d\xi\,dz \tag{67}$$

However, when z is a mere parameter or a constant, we have

$$g(x,z)=\int_{-\infty}^{\infty} l(x-\xi,z)\,\rho(\xi,z)\,d\xi \tag{68}$$

or

$$g(x)=\int_{-\infty}^{\infty} l(x-\xi)\,\rho(\xi)\,d\xi. \tag{68$'$}$$

In the following, the corresponding quantity $\tilde{\Delta} g(x)$ to the quantity $\Delta\rho(x,z)$ is considered. The quantity $\Delta\rho\,(x, z)$ is given by

$$\Delta\rho\,(x, z)=\int_{-\infty}^{\infty} k\,(x-\xi)\;\rho\,(\xi, z)\;d\xi,$$

where $k\,(x-\xi)$ is the Fourier transform of the characteristic function $K\,(\omega)$ and $K\,(\omega)$ is an arbitrary characteristic function of the running average method. Then $\tilde{\Delta g}\,(x)$ is given by

$$\begin{aligned}\tilde{\Delta g}\,(x) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\,(x-\xi, z)\;\Delta\rho\,(\xi, z)\;d\xi\,dz \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\,(x-\xi, z)\;k\,(\xi-\xi^*)\;\rho\,(\xi^*, z)\;d\xi^*\,d\xi\,dz \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\,(x-\xi, z)\;\rho\,(\xi-\xi^*, z)\;k\,(\xi^*)\;d\xi\,dz\,d\xi^*.\end{aligned}$$

Here,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\,(x-\xi, z)\;\rho\,(\xi-\xi^*, z)\;d\xi\,dz=g\,(x-\xi^*).$$

Then we have

$$\begin{aligned}\tilde{\Delta g}\,(x) &= \int_{-\infty}^{\infty} g\,(x-\xi^*)\;k\,(\xi^*)\;d\xi^* \\ &= \int_{-\infty}^{\infty} k\,(x-\xi^*)\;g\,(\xi^*)\;d\xi^*=\Delta g\,(x).\end{aligned}$$

Therefore, it is concluded that $\Delta g\,(x)$ is produced by $\Delta\rho\,(x, z)$ and, reversely speaking, when $\rho\,(x, z)$ is known, the corresponding quantity to $\Delta g\,(x)$ is given by $\Delta\rho\,(x, z)$ for arbitrary value of z. Here, it should be paid attention that we can not calculate $\Delta\rho\,(x, z)$ from $\Delta g\,(x)$ unless the functional form of $\rho\,(x, z)$ is known. However,in the case of eq. (68) or eq. (68)′ it is proved that $\Delta\rho\,(x)$ can be calculated uniquely from $\Delta g\,(x)$ or $g\,(x)$ as follows:

Now, when the kernel of the inverse transform of the integral transform expressed by eq. (68) is denoted by the symbol $l^{-1}\,(x, \xi)$, we have

$$\begin{aligned}\rho\,(x) &= \int_{-\infty}^{\infty} l^{-1}(x, \xi)\;g\,(\xi)\;d\xi \\ &= \int_{-\infty}^{\infty} l^{-1}(x-\xi)\;g\,(\xi)\;d\xi.\end{aligned} \tag{69}$$

By using above relation the corresponding quantity $\Delta\rho\,(x)$ to $\Delta g\,(x)$ is calculated as below.

$$\begin{aligned}\tilde{\Delta\rho}\,(x) &= \int_{-\infty}^{\infty} l^{-1}(x-\xi)\;\Delta g\,(\xi)\;d\xi \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l^{-1}(x-\xi)\;k\,(\xi-\xi^*)\;g\,(\xi^*)\;d\xi^*d\xi \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l^{-1}(x-\xi)\;g\,(\xi-\xi^*)\;k\,(\xi^*)\;d\xi\,d\xi^* \\ &= \int_{-\infty}^{\infty} k\,(x-\xi)\;\rho\,(\xi)\;d\xi=\Delta\rho\,(x).\end{aligned} \tag{70}$$

Therefore, the physical quantity corresponding to $\Delta g\,(x)$ is given by $\Delta\rho\,(x)$, and this quantity can be calculated from $\Delta g\,(x)$ or $g\,(x)$ directly.

From the considerations mentioned above, it becomes clear that the residual gravity obtained by the use of the running average method is produced by the distribution of the corresponding residual density which is obtained by applying the present method to the subterranean density distribution at arbitrary depth.

Further, if the hypothesis of the condensation surface was used, the distribution of the residual density on this condensation surface can be calculated from the distribution of the residual gravity, and in this time parameter z represents the depth of this condensation surface.

V. Detections used in Gravity Prospecting

As already pointed out, there are many methods which purposes are to remove the larger regional background anomalies and the smaller anomalies (noises) and to bring out the significant features. However, these methods have some defects, e. g., complexity of an actual calculation, insufficiency of removing the noises and indistinctness of the physical meaning of detected anomalies and so on. Then the writer considers in detail the desirable natures of the method to obtain the significant features of gravity anomalies by removing the regional gravity and noises. As the results of this consideration, he concludes that the desirable natures of the method of gravity analysis are as follows:

(1) The method should be able to well point out only the presence of the anomalies of objective scale, but their magnitude and their shapes.

(2) It is desirable that the physical meaning of the anomalies detected is distinct.

(3) It is desirable that the qualitative interpretation of the anomalies obtained can be easily performed without any mathematical and physical knowledges of high degree.

(4) The anomalies detected must become the object of the theoretical consideration.

(5) It is necessary that the calculation of obtaining the objective anomalies can be performed simply and easily.

(6) The economy of the measurement points should be considered.

(7) It is necessary that the relation of the two anomalies of the different scale can be considered theoretically.

It may be able to understand easily from the discussion in the previous chapters that the running average method has the several characters generally of the desirable natures mentioned above. However, all the detections of the present method have not always all the characters mentioned above. That is, when we make much of the condition (1), we have only the two excellent detections of their selection coefficients $K_{1,3}(\lambda)$ and $K_{1,4}(\lambda)$ for anomalies of small scale. Fig. 9 shows the comparison of the characteristics between the Henderson-Zietz's formula and the normal detection in one dimensional case, where the former was used in our Geological Survey in the past. As shown in Fig. 9, it is evident that the influence of noises to the significant features in the use of the normal detection is very

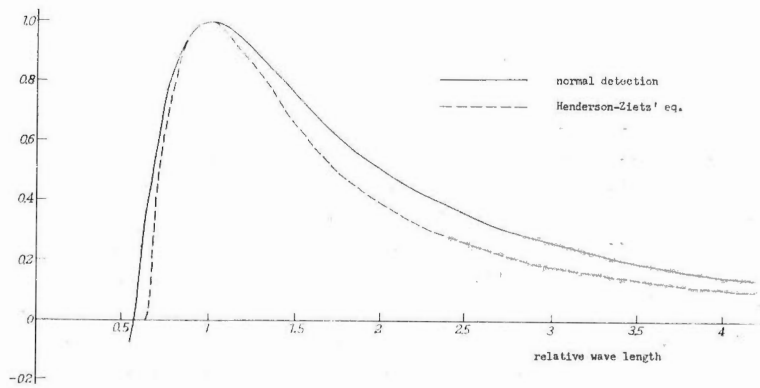

Fig. 9-a Comparison of the characteristics between the normal detection and Henderson-Zietz's formula (principal part)

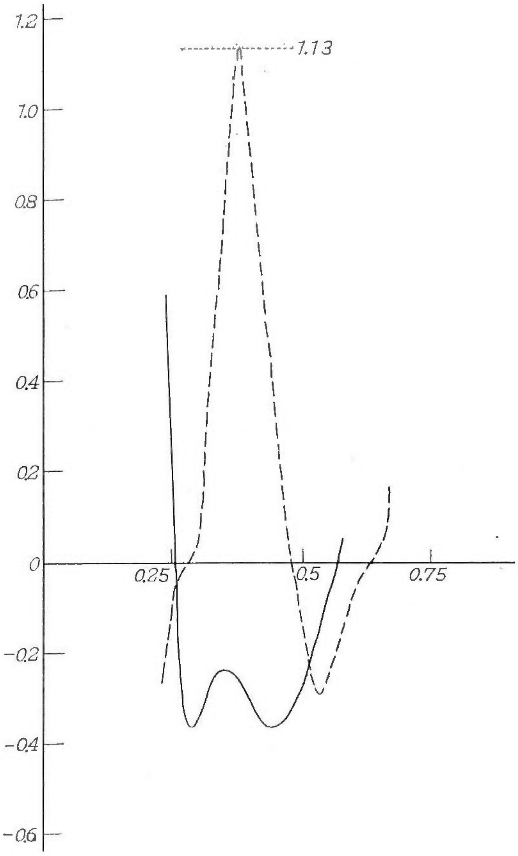

Fig. 9-b ditto (subordinate part)

smaller than that in the use of the Henderson-Zietz's formula.

Further, when the conditions (6) and (7) are considered, particularly the condition (7), we can not help but adopt the normal detection. That is, when the anomalies of larger scale than the scale of the normal structure is considered, the bi-structural detection should be adopted for the following reason.

From the definition of the running average method it is evident that the gravity anomaly composed of the normal structure and the bi-structure is the same as the residual gravity obtained by the detection with the selection coefficient $K_{1,7}(\lambda)$, i.e.,

$$\Delta_{1,7}\, g\,(x) = \Delta_{1,3}\, g\,(x) + \Delta_{3,7}\, g(x).$$

Therefore, the consideration of the gravity anomalies is performed in detail by dividing a gravity anomaly into four parts of different scale, namely, the noise structure $\Delta_{0,1}\, g\,(x)$, the normal structure $\Delta_{1,3}\, g\,(x)$, the bi-structure $\Delta_{3,7}\, g\,(x)$ and the regional background anomaly $\bar{g}^7\,(x)$. As the result of such a consideration, the underground structures, particularly their significant features, may be clarified.

If any other detection is adopted instead of the bi-structural detection, e.g., the detection with its selection coefficient $K_{2,7}(\lambda)$, we shall be sorely perplexed. Because, this detection has the excellent characteristic as before pointed out, but the anomaly obtained by this detection includes the quantity $\Delta_{2,3}\, g\,(x)$. That is,

$$\Delta_{1,3}\, g\,(x) + \Delta_{2,7}\, g\,(x) = \Delta_{1,7}\, g\,(x) + \Delta_{2,3}\, g\,(x).$$

Therefore, when the subterranean structures based on $\Delta_{1,3}\, g\ (x)$ and $\Delta_{2,7}\, g\ (x)$ are presumed, we shall make a mistake that we add the overmuch excess mass based on $\Delta_{2,3}\, g\ (x)$.

Moreover, if we adopt any other detection, e. g., its selection coefficient is $K_{3,10}\ (\lambda)$, what problem arise? In this case $(2\beta+1)/(2\alpha+1)=3$, then the detection has the excellent characteristic and at this time there is no defect as pointed out above. But in this case two problems occur, namely, one is the problem with regard to the condition (6) and the other is the problem that the scale of the anomaly obtained seems like somewhat large. Particularly, the former is important in the practice.

Further, the reason of that the detection with its selection coefficient $K_{1,4}\ (\lambda)$ dose not be adopted is as follows:

(a) To adopt this detection is in conflict with the condition (6).

(b) The difference between $K^*_{1,4}(\lambda^*)$ and $K^*_{1,3}(\lambda^*)$ can be neglected practically.

(c) Moreover, when an anomaly of larger scale is considered, we must take a detection of $\alpha=4$ and $\beta=11\sim15$, then this is also in conflict with the condition (6).

Thus the reasons of the adoption of the normal detection and the bi-structural detection become clear.

It is sure, if we do not consider very much the influence of noises and if we make much of the condition (6), we can adopt some other detection, e. g., detection of its selection coefficient $K_{1,2}\ (\lambda)$ for anomalies of small scale and detection with $K_{2,5}\ (\lambda)$ for anomalies of larger scale.

VI. Quantitative Interpretation of Residual Gravity

VI. 1 Fundamental Theory

As already mentioned in chapter I, it is possible to calculate a gravity value on the earth surface from the subterranean density distribution, but on the contrary it is impossible to calculate theoretically the subterranean density distribution from the gravity distribution on the earth surface without any assumption of density. However, practically, the solution of this problem is expected from the requirement in the prospecting of underground resources. And by this reason many efforts to calculate the subterranean density distribution from the given gravity anomalies have been done by using various assumptions for models of the density distribution. But so far, in the presence we have no established method of the interpretation of the gravity anomaly.

The residual gravity obtained by the writer's method means deviation from average tendency of large scale, and its causes are undulation of comparatively small scale of the basement, massive bodies of different material (e. g., rock salt dome) and geological structures in thick sedimentary formations (e. g., anticline, fault) etc. The last cause of these has generally small influence to the gravity in spite of the significant meaning in the geophysical prospecting, and then with this reason the various methods to detect local and weak anomalies have been investigated. However, as pointed out above, the established method of the quantitative interpretation of the residual gravity detected is not present.

In this chapter, he will explain the method which may be applicable for the case that the residual gravity obtained by his method is caused by the geological structures in the sedimentary formations thick enough.

In the following, the two dimensional case is considered. Then, let the subterranean density distribution be independent of y-coordinate, and suppose the z-axis is taken vertically. So the density distribution is represented as

$$\rho\ (x, z) = \bar{\rho}\ (z) + \tilde{\rho}\ (x, z), \tag{71}$$

where $\rho\ (x, z)$ is the density at point $P\ (x, z)$ in the subsurface and $\bar{\rho}\ (z)$ is average density at a depth z. Accordingly, $\tilde{\rho}\ (x, z)$ means a deviation from the average density at the depth z. Here, put

$$\tilde{\rho}\ (x, z) = \rho_2\ (z)\ \rho_1\ (x, z) \tag{72}$$

and denote the spectrum of $\rho_1\ (x, z)$ for distance variable x by symbol $R_1\ (\omega, z)$, i. e.,

$$R_1\ (\omega, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_1\ (x, z)\ e^{i\omega x}\, dx. \tag{73}$$

The residual density at the depth z corresponding to the residual gravity is given by

$$\varDelta \rho\ (x, z) = \varDelta \tilde{\rho}\ (x, z) = \rho_2\ (z)\ \varDelta \rho_1\ (x, z), \tag{74}$$

where $\varDelta$ denote a operator of any detection of the present method, e. g., normal detection, bi-structural detection, etc. The quantity $\varDelta \rho_1\ (x, z)$ is given by the use of eq. (73) as

$$\begin{aligned} \varDelta \rho_1\ (x, z) &= \int_{-\infty}^{\infty} k\ (x-\xi)\ \rho_1\ (\xi, z)\ d\xi \\ &= \int_{-\infty}^{\infty} K\ (\omega)\ R_1\ (\omega, z)\ e^{i\omega x}\, d\omega. \end{aligned} \tag{75}$$

Gravity anomaly on the earth surface caused by infinitely extended horizontal plate of infinitesimal thickness $d\xi$ with the density distribution $\varDelta \tilde{\rho}\ (\xi, \zeta)$ at the depth ζ is

$$d \varDelta g\ (x, \zeta) = 2\pi\gamma\rho_2\ (\zeta)\ d\zeta \int_{-\infty}^{\infty} K\ (\omega)\ R_1\ (\omega, \zeta)\ e^{-|\omega|\zeta}\, e^{i\omega x}\, d\omega,$$

where γ is the universal constant of gravitation. Then the residual gravity is expressed by the following analytical representation.

$$\varDelta g\ (x) = 2\pi\gamma \int_{-\infty}^{\infty} K\ (\omega)\ e^{i\omega x} \left[\int_0^{\infty} \rho_2\ (\zeta)\ R_1\ (\omega, \zeta)\ e^{-|\omega|\zeta}\, d\zeta \right] d\omega. \tag{76}$$

The integration in the bracket of above equation can not be performed unless the functional forms for the variable z of $\rho_2\ (z)$ and $R_1\ (\omega, z)$ is known. Then the following relations are assumed.

$$\left\{ \begin{aligned} &\rho_2\ (z) = 1 - e^{-w\,(z+a)}, & (77) \\ &\nabla_k^2 \rho_1\ (x, z) \equiv \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{k^2 \partial z^2} \right] \rho_1\ (x, z) = 0, & (78) \end{aligned} \right.$$

where w, α and k are the constants and the symbol $\varDelta_k^2$ means the generalized Laplacian.

Next, for a while the above assumptions will be considered.

For the assumption of eq. (77), we must call to mind that in many cases there is the well-known relation between density of sedimentary rock and its buried depth, that is,

$$\bar{\rho}\ (z) = A + B\ (1 - e^{-wz}), \tag{79}$$

where A and B are the constants and other symbols are already explained. Therefore, a density at arbitrary subsurface point can be expressed as

$$\rho\ (x, z) = A + B'\ (1 - e^{-wz}), \tag{80}$$

where B' is a function of variables x and z, i. e.,

$$B' = B'\ (x, z).$$

If the initial density distribution expressed by eq. (80) is kept unchangeable for the erosive action, the density distribution at the time when the erosion has acted untill the depth α (see Fig. 10) is expressed by the following equation for the new earth surface.

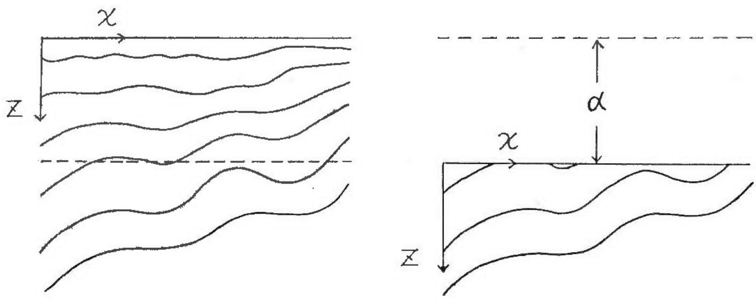

Fig. 10 Erosive action

$$\rho\ (x, z) = A + B'\ \{1 - e^{-w\,(z+\alpha)}\},\quad B' = B'\ (x, z+\alpha). \tag{81}$$

In eq. (79), the constant A means the average density at the earth surface, but the constant A of eq. (81) has not such clear meaning and this should be calculated from the data of the density of rocks together with the value of α. From the above consideration it is concluded that the assumption of eq. (77) is reasonable.

On the contrary, the hypothesis of eq. (78), namely, the assumption on the function $B'\ (x, z)$ in eq. (80), is including problems. This matter is caused by the fact that the relation between the force by which various subterranean structures are produced and the density of sedimentary rocks is not known entirely. However, it is supposed that when geological structures are formed, density of rocks at any depth will be changed its value with their structural variation to horizontal and vertical directions, and the density variations of each direction is not independent each other. Then the hypothesis eq. (78) can be understood as the mathematical expression of the consideration mentioned above. And it is supposed that the constant k is the parameter determined by the dynamical characteristics of the tectogenesis and by the natures of sedimentary formations.

Kato[24] assumed the density distribution in his paper as

$$\rho = \bar{\rho} + \sum X_m \begin{matrix}\cos\\ \sin\end{matrix}\ a_m x,$$

where

$$X_m = e^{-\lambda z} D_m + (1 - e^{-\lambda z})\ e^{-\lambda_m z} C_m$$

and he took 0.5 km^{-1} as the value of λ, i.e., w in the present paper, from many data of sedimentary rocks at various depth. And he studied on the two examples about three cases in which he assumed $\lambda_m = 0$, $= \lambda$, and $= a_m$ respectively. As the results, he concluded that in the case of $\lambda_m = a_m$ good result is obtained. This assumption of λ_m is the same to the case of $k = 1$ in eq. (78).

In this paper the assumptions of eq. (77) and eq. (78) are adopted by considering the Kato's result. This matter is considered unaboidable in the present stage as far as we have little data for a variation of underground density and we can not overcome the theoretical difficulty of the problem.

From the assumption of eq. (78) the following general solution of $\rho_1\ (x, z)$ is obtained.

$$\rho_1\ (x, z) = \int_{-\infty}^{\infty} R_1(\zeta) e^{-kz|\varphi|z} e^{i\varphi x} d\varphi,$$

where $R_1\ (\varphi)$ is an arbitrary function of variable φ. Therefore, we have

$$R_1\ (\omega, z) = e^{-k|\omega|z}\ R_1(\omega). \tag{82}$$

Then the integration in the bracket of eq. (76) can be carried out by using eq. (77)

and eq. (82), i. e.,

$$\int_0^{\infty} \rho_2(z)\ R_1(\omega, \zeta)\ e^{-|\omega|\zeta} d\zeta = \left[\frac{1}{(1+k)|\omega|} - \frac{e^{-wa}}{\{(1+k)|\omega|+w\}}\right] e^{-k|\omega|a} R_1(\omega).$$

If α is comparatively small, the right hand of above equation becomes

$$\frac{w\{1+(1+k)\alpha|\omega|\}}{(1+k)|\omega|\{(1+k)|\omega|+w\}} e^{-k|\omega|a} R_1(\omega).$$

Therefore, eq. (76) becomes

$$\Delta g(x) = 2\pi\gamma \int_{-\infty}^{\infty} \frac{w\{1+(1+k)\alpha|\omega|\}}{(1+k)|\omega|\{(1+k)|\omega|+w\}} e^{-k|\omega|a} K(\omega)\ R_1(\omega)\ e^{i\omega x} d\omega. \tag{83}$$

On the other hand

$$\Delta g(x) = \int_{-\infty}^{\infty} \Delta G(\omega)\ e^{i\omega x} d\omega,$$

so we have

$$R_1(\omega) = \frac{1}{2\pi\gamma} \frac{(1+k)|\omega|\{(1+k)|\omega|+w\}}{w\{1+(1+k)\alpha|\omega|\}} e^{k|\omega|a} \frac{\Delta G(\omega)}{K(\omega)}. \tag{84}$$

Thus a residual density at arbitrary subsurface point is obtained as follows:

$$\Delta\rho(x, z) = \frac{1-e^{-w(z+a)}}{2\pi\gamma} \int_{-\infty}^{\infty} \frac{(1+k)|\omega|\{(1+k)|\omega|+w\}}{w\{1+(1+k)\alpha|\omega|\}} e^{-k|\omega|z} \Delta G(\omega)\ e^{i\omega x} d\omega. \tag{85}$$

When the value of α is 0 or small enough, above equation gives

$$\Delta\rho(x, z) = \frac{1-e^{-wz}}{2\pi\gamma} \int_{-\infty}^{\infty} (1+k)|\omega|\{(1+k)|\omega|+w\} e^{-k|\omega|z} \Delta G(\omega)\ e^{i\omega x} d\omega. \tag{86}$$

These equations, eq. (85) and eq. (86), indicate what a filtering action the density distribution has. That is, analogously saying, when the input is the residual gravity of its spectrum $\Delta G(\omega)$ and the output is the residual density given by eq. (85) or eq. (86), the frequency responses of the filters of respective cases are expressed as follows respectively:

$$F_a(\omega, z) = \frac{1-e^{-w(z+a)}}{2\pi\gamma w} \frac{(1+k)|\omega|\{(1+k)|\omega|+w\}}{\{1+(1+k)\alpha|\omega|\}} e^{-k|\omega|z}), \tag{87}$$

$$F(\omega, z) = \frac{1-e^{-wz}}{2\pi\gamma w} (1+k)|\omega|\{(1+k)|\omega|+w\} e^{-k|\omega|z}. \tag{88}$$

VI. 2 Characteristics of "the Density-Spacial Filter"

VI. 2. 1 Characteristics of $F(\omega, z)$

Thus, it becomes very important that the characteristics of the frequency (wave number) responses mentioned above are considered. So, at first, the case of eq. (88) will be examined for its simplicity.

In eq. (88) the part of the function of variable z, i. e.,

$$Z(\omega, z) = (1-e^{-wz})\ e^{-k|\omega|z} \tag{89}$$

is considered. This function has the maximum value Z_M for arbitrary constant value of ω at the depth

$$z_M = \frac{1}{w} \log\left(1+\frac{w}{k|\omega|}\right). \tag{90}$$

In Fig. 11 the function $Z(\omega, z)$ are illustrated for several values of ω. From Fig. 11 it is evident that this function changes its value gradually with depth in considerably broad range about the depth z_M. Therefore, it is supposed that a gravity anomaly of wave number $\omega/2\pi$ is caused by a density anomaly of same wave number in comparatively wide range of before and behind the depth z_M (the depth of maximum amplitude) which is determined by the wave number $\omega/2\pi$ and at which the density anomaly has the maximum amplitude. Here, we must pay

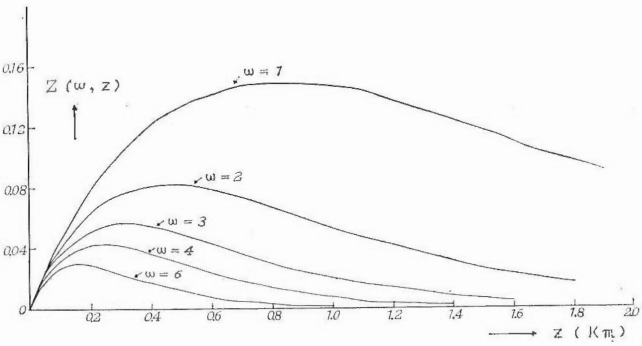

Fig. 11 Curves of the function Z (ω, z) for several values of ω (k=1)

attention to the meaning of k. That is, as shown in eq. (90), the depth z_M is depend on the constant k, and the depth of maximum amplitude z_M of the wave number $\omega/2\pi$ in the case of $k=1$ is equal to that of the wave number $\frac{\omega}{2\pi}/k$ for the case of arbitrary taken value of k.

Eq. (90) becomes for comparatively large values of ω

$$z_M \doteqdot 1/k\,|\omega|\,. \tag{90}'$$

This is important relation for our purpose. That is, we are now considering the residual gravity, and in many cases we take the spacing of measurement as $S=$ 500m or $S=250$m, so the predominant wave number of normal structure of gravity becomes $\omega/2\pi=1/\pi\ km^{-1}$ or $\omega/2\pi=2/\pi\ km^{-1}$ respectively. On the other hand, $w=0.5$ km^{-1} as pointed out by Kato,[24] therefore, we can accept usually the approximate relation of eq. (90)'. And from this relation we can estimate "the depth of the presence" of the residual density, and we can modify this depth by means of adequate value of k.

From the matter mentioned above it is presumed that k is the parameter which may be dependent on "the depth of action" of the force acting in the tectogenesis, on the magnitude of this force, the scale of this force, etc. or on the nature of sedimentary formations and their history.

Next, the character of the function F (ω, z) will be investigated. Then the following function is considered.

$$W(\omega, z)=(1+k)|\omega|\{(1+k)|\omega|+w\}\,e^{-k|\omega|z}. \tag{91}$$

Here, at first the constant w in eq. (91) is neglected for the simplicity of the consideration. That is, the function

$$W_0(\omega, z)=(1+k)^2\omega^2 e^{-k|\omega|z} \tag{92}$$

is considered. This function has the maximum value $W_{0M}(z)$ for the constant value of z at

$$\overset{\circ}{\omega}_{\rm Max}=2/kz. \tag{93}$$

Eq. (92) is also expressed by the form

$$W_0(\omega, z)=W_0(\omega_*, z)=W_{0M}(z)\cdot W_0^*(\omega_*), \tag{94}$$

where

$$W_{0M}(z)=\left\{\frac{2(1+k)}{e}\right\}^2\cdot\frac{1}{(kz)^2} \tag{95}$$

and

$$W_0^*(\omega_*)=\omega_*^2\,e^{-2(\omega_*-1)}. \tag{96}$$

The function $W_0^*(\omega_*)$ is independent of depth and has the characteristic shown in Fig. 12. Therefore, the function $W_0(\omega, z)$ has the maximum value inversely proportional to the square of depth z at the wave number $\omega^{\circ}_{\text{Max}}/2\pi$ (maximum detected wave number) determined by a depth, and its value approaches to 0 monotoniquely for a wave number larger or smaller than "the maximum detected wave number", $\omega^{\circ}_{\text{Max}}/2\pi$, but types of filter curves for arbitrary values of z are strictly identical.

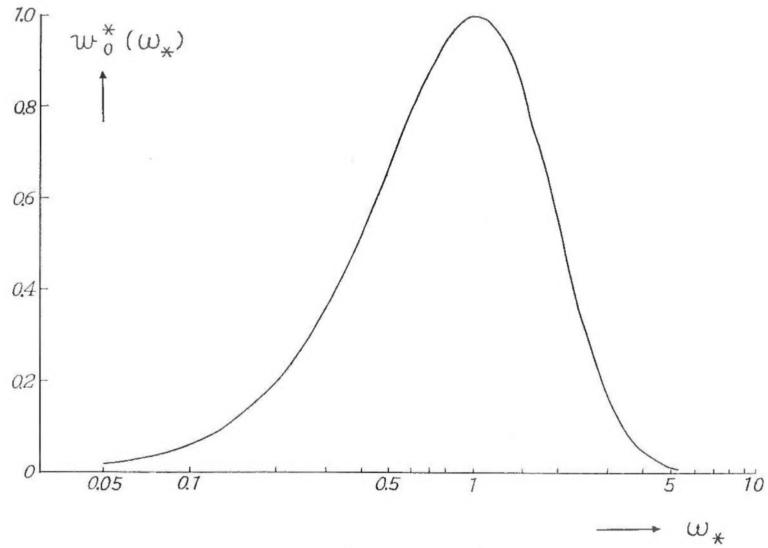

Fig. 12 Curve of the function $W_0{}^*(\omega_*)$

Moreover, the influence of the parameter k is similar to the previous case as shown in eq. (93), and this is very interesting matter. That is, from the deformed relations,

$$\left.\begin{array}{l} z_M = \dfrac{1}{k\omega} \\ \dfrac{1}{2}z = \dfrac{1}{k\omega^{\circ}_{\text{Max}}} \end{array}\right\} \quad \text{or} \quad \left.\begin{array}{l} \omega = \dfrac{1}{kz_M} \\ \dfrac{1}{2}\omega^{\circ}_{\text{Max}} = \dfrac{1}{kz} \end{array}\right\}, \tag{97}$$

it is able to say that the depth of maximum amplitude of a density variation of a wave number $\frac{\omega}{2\pi}$ is a half of the depth at which the gravity variation of the same wave number is subjected to the maximum detection. or that a wave number of which the amplitude of a density variation becomes maximum at a depth z_M is a half of the maximum detected wave number at the same depth.

Thus we can presume a structure of residual densities corresponding to residual gravities of comparatively small scale. For example, a residual density of the normal structure will distribute in comparatively broad range of depth about the depth S/k so far as the spacing S is taken comparatively small, because the normal structure of gravity is the structure with its predominant wave length of about 6S, i. e., predominant wave number of about $1/12\pi S$. And the depth at which this density variation becomes predominant than other scale variation will be estimated of about $z=2S/k$. And at this depth a density variation of its wave number larger than about three times of $1/12\pi S$ or less than about fifth of $1/12\pi S$ can be neglected. So it is supposed that, roughly saying, the figure of residual density may be similar to the figure of the residual gravity.

In the consideration mentioned above, the constant w in the expression of the function $W(\omega, z)$ was neglected, but in the following the constant w will be considered.

The maximum point of the function $w(\omega, z)$ for a constant value of z is given by

$$\omega_{\text{Max}} = \frac{1+\sqrt{1+k^2z^2w^2/2^2(1+k)^2} - kzw/2(1+k)}{kz}. \tag{98}$$

This equation becomes simpler in the present case, for depth considered is at most a few kilometers. That is,

$$\omega_{\text{Max}} \doteqdot \frac{2}{kz}\left\{1 - \frac{kz}{8(1+k)} + \frac{(kz)^2}{8^2(1+k)^2}\right\}.$$

Here, a magnitude of $kz/8(1+k)$ is estimated for a few cases. When $k=1$ and $z=2$ (km), that value is equal to 1/8. And when $k=2$ and $z=2$ (km), that value becomes 1/6. Therefore, in practice the following approximate relation can be used.

$$\omega_{\text{Max}} \doteqdot \frac{2}{kz}\left\{1 - \frac{kz}{8(1+k)}\right\}. \tag{99}$$

Further, for small values of z it becomes

$$\omega_{\text{Max}} \doteqdot \frac{2}{kz} = \omega^0_{\text{Max}}$$

On the other hand, the maximum value of $W(\omega, z)$ is given by

$$W_M(z) \doteqdot \left\{1 + \frac{kz}{4(1+k)}\right\} \cdot W_{0M}(z). \tag{100}$$

Therefore, the influence of the constant w to the maximum value of the function is somewhat larger than that to the maximum point.

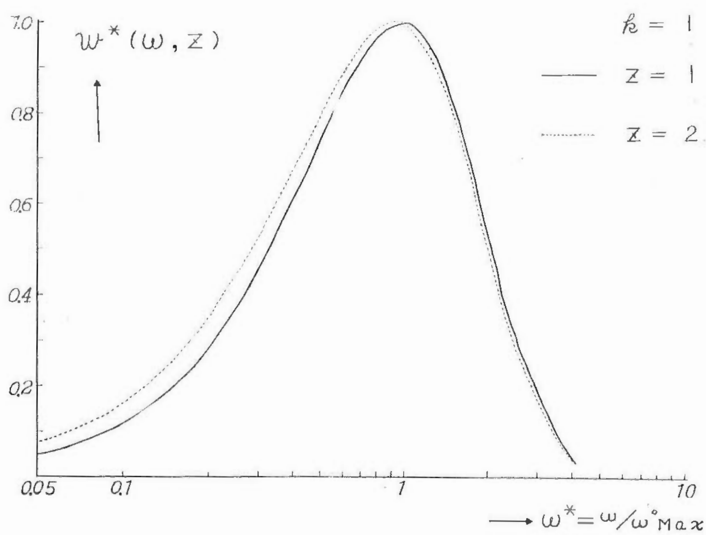

Fig. 13 Curves of $w(\omega, z)$ for different values of z ($W^*(\omega, z) = C_2 W(\omega, z)$, where C_2 is the normalizing constant)

As the conclusion, it is supposed that the difference between $W(\omega, z)$ and $W_0(\omega, z)$ is a little as far as consider a residual gravity of usual scale, and accordingly, in usual case, we can use

$$\left\{1+\frac{kz}{4(1+k)}\right\}W_0(\omega, z)$$

little precisely or merely $W_0(\omega, z)$ instead of the function $W(\omega, z)$.

In Fig. 13 this difference is illustrated concretely for a few cases.

VI. 2. 2 Characteristics of $F_\alpha(\omega, z)$

In the following, the function $F_a(\omega, z)$ will be considered. The constant w is also neglected in the middle bracket $\{\ \}$ in eq. (87) for the simplicity of the consideration. That is,

$$W_{0a}(\omega, z)=\frac{1-e^{-w(z+a)}}{2\pi\gamma w}\cdot\frac{(1+k)^2\omega^2}{1+\alpha(1+k)|\omega|}e^{-k|\omega|z}. \tag{101}$$

Here, at first the function

$$Z_a(\omega, z)=\{1-e^{-w(z+a)}\}e^{-k|\omega|z} \tag{102}$$

is considered as before. Then the maximum point of this function for a constant value of ω is given by

$$z_{aM}=\frac{1}{w}\log\left\{1+\frac{w}{k|\omega|}\right\}-\alpha, \tag{103}$$

that is,

$$z_{aM}+\alpha=\frac{1}{w}\log\left\{1+\frac{w}{k|\omega|}\right\}. \tag{103'}$$

The left hand side of above equation means the depth of maximum amplitude of a density variation of wave number $\omega/2\pi$ for the earth surface before the erosion. Therefore, the depth of maximum amplitude of a density variation of wave number $\omega/2\pi$ for the former earth surface is retained to constant throughout the erosion.

Next, the function

$$W_{0,a}(\omega, z)=\frac{(1+k)^2\omega^2}{1+\alpha(1+k)|\omega|}e^{-k|\omega|z} \tag{104}$$

is considered. This function has the maximum value for the constant value of z at the point $\omega^\circ_{a\text{Max}}$ on the ω-axis which value is calculated by the equation

$$z=\frac{1}{k\omega^\circ_{a\text{Max}}}\cdot\frac{\alpha(1+k)\omega^\circ_{a\text{Max}}+2}{\alpha(1+k)\omega^\circ_{a\text{Max}}+1}. \tag{105}$$

Then we have

for $z \ll 1$,

$$\omega^\circ_{a\text{Max}} \approx 1/kz,$$

for $z \gg 1$,

$$\omega^\circ_{a\text{Max}} \sim 2/kz,$$

and for general value of z

$$\frac{1}{kz} \leq \omega^\circ_{a\text{Max}} \leq \frac{2}{kz}.$$

In Table 4, 2π times wave numbers of the variation and corresponding depths for the earth surface before the erosion are shown for a few values of α. And in 2nd column of this table the depths corresponding to each wave number in the case of $\alpha=0$, i. e., the erosive action was not present, are tabulated.

Thus, it is concluded that the influence of the erosive action is not present on the depth of maximum amplitude but that erosive action influences to the depth correspond to the maximum detected wave number. However, this matter was already expected in the previous consideration. That is, from the previous consideration it is recognized that the function $Z(\omega, z)$ indicates the model of the

Table 4 Depths for the earth surface before the erosion at which the characteristic function $w_{0,\alpha}$ (ω, z) has the central wave number$\omega^{o}_{\alpha \mathrm{Max}}/2\pi$

$\omega^{o}_{\alpha \mathrm{Max}}$	$2/\omega^{o}_{\alpha \mathrm{Max}}$	$z+\alpha$		
		$\alpha=0.25$	$\alpha=0.5$	$\alpha=1.0$
0.2	10.0	9.795	9.667	9.600
0.5	4.0	3.850	3.830	4.000
0.8	2.5	2.393	2.444	2.731
1.0	2.0	1.917	2.000	2.333
1.5	1.3	1.298	1.433	1.833
2.0	1.0	1.000	1.170	1.600
3.0	0.6	0.717	0.917	1.381
4.0	0.5	0.583	0.800	1.270

distribution of the density anomaly and the function $W(\omega, z)$ is the characteristic function of "the density-spacial filter" which is determined by the presence of the space and the density distribution, then consequently it is presumed that the influence of the erosive action is not present in the former but may complicate the characteristic of the filter.

In order to investigate the characteristic of the function $W_{0,\alpha}(\omega, z)$ the case of $k=1$, $\alpha=0.5$ is considered in the following, i. e., in this case the characteristic function becomes

$$W_{0,0.5}(\omega, z)=\frac{4\omega^2}{1+|\omega|}e^{-|\omega|z}. \tag{106}$$

In Fig. 14 the characteristic curve of this case for $z=1$ is shown by dotted line. And the curve of solid line is that of the same function but is represented by the different expression, i. e.,

$$W^{*}_{0,0.5}(\omega^{**}, 1)=4C\frac{\omega^2_{\mathrm{Max}}\,\omega^{**2}}{1+\omega_{\mathrm{Max}}\,\omega^{**}}\cdot e^{-\omega_{\mathrm{Max}}\omega^{**}}, \tag{107}$$

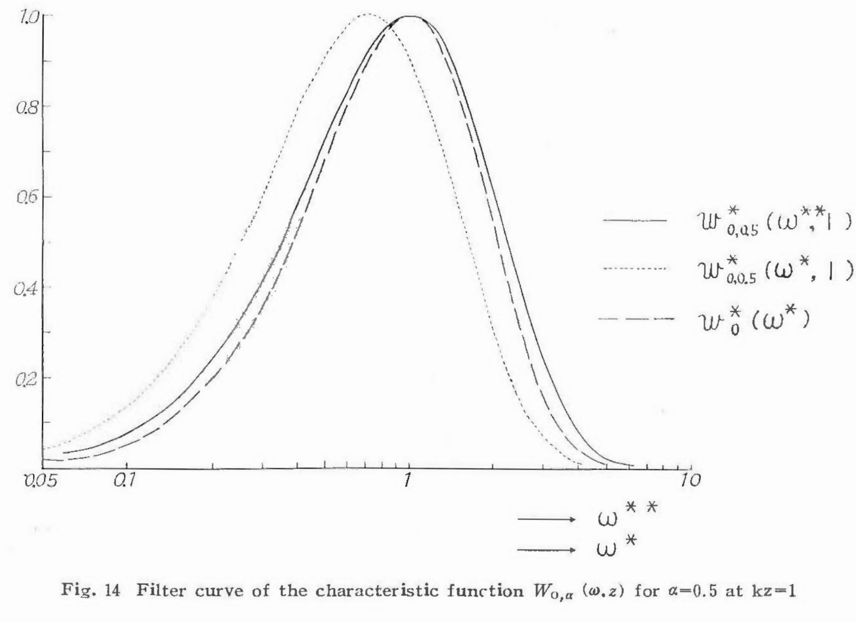

Fig. 14 Filter curve of the characteristic function $W_{0,\alpha}$ (ω, z) for $\alpha=0.5$ at kz=1

where $\omega_{Max}=1.4286$, and $C=1.2411$ is the normalizing constant. Moreover, the curve of dashed line is the characteristic curve of $W_0^*(\omega_*)$. From Fig. 14 it becomes clear that the position of the maximum point of $w_{0,0.5}(\omega, z)$ transit to the direction of small wave number, and the character of high cut becomes strong. But the curves of $W_{0,0.5}^*(\omega^{**}, 1)$ and $W_0^*(\omega_*)$ are nearly identical, and we can also see the similar relations for other arbitrary values of z. Such character of the density-spacial filter indicates that the erosive action dose not give the essential influence to the character of this filter.

VI. 2. 3 Characteristics of $K_z(\omega)$

Hitherto the characteristics of the density-spacial filter have been considered, but in the following the wave number response of the case in which the residual density distribution is calculated directly from the gravity distribution will be investigated.

Eq. (86) can be deformed by using the relation $\Delta G(\omega)=K(\omega)\,G(\omega)$ as

$$\Delta\rho(x, z)=\frac{1-e^{-z/2}}{\pi\gamma}\int_{-\infty}^{\infty}(1+k)^2\omega^2 e^{-k|\omega|z}\cdot K(\omega)\cdot G(\omega)\,e^{i\omega x}d\omega. \qquad (108)$$

Then when the function $K_z(\omega)$ is defined by

$$K_z(\omega)=K(\omega)\cdot W_0(\omega, z), \qquad (109)$$

eq. (108) becomes

$$\Delta\rho(x, z)=\frac{1-e^{-z/2}}{\pi\gamma}\int_{-\infty}^{\infty}K_z(\omega)\cdot G(\omega)\,e^{i\omega x}d\omega. \qquad (110)$$

This is the equation to calculate the residual density directly from the gravity distribution on the earth surface. Then the wave number response of this case becomes

$$\frac{1-e^{-z/2}}{\pi\gamma}\cdot K_z(\omega).$$

Eq. (109) is also expressed by the normalized form of

$$K_z^*(\omega^*)=CW_{0,M}(z)\cdot W_0^*(\omega_*)\,K^*(\omega^*), \qquad (111)$$

where C is the normalizing constant and

$$\omega_*=\omega/\omega^0{}_{Max}, \quad \omega^*=\omega/\omega_M$$

and ω_M denotes the central wave number of the characteristic function $K(\omega)$. In order to investigate this combined characteristic function, the following three cases, i. e., cases of $kz=1/\omega_M$, $=2/\omega_M$ and $=4/\omega_M$, will be considered.

(a) The case of $kz=1/\omega_M$

In this case we have $\omega^0{}_{Max}=2\omega_M$, so

$$\omega_*=\frac{1}{2}\omega^*.$$

Then eq. (111) gives

$$\left[K_z^*(\omega^*)\right]_{kz=1/\omega_M}=CW_{0,M}\left(\frac{1}{\omega_M}\right)W_0^*\left(\frac{\omega^*}{2}\right)K^*(\omega^*).$$

(b) The case of $kz=2/\omega_M$

In this case we have $\omega^\circ{}_{Max}=\omega_M$, so

$$\omega_*=\omega^*.$$

Then eq. (111) gives

$$\left[K_z^*(\omega^*)\right]_{kz=2/\omega_M}=CW_{0,M}\left(\frac{2}{\omega_M}\right)W_0^*(\omega^*)\,K^*(\omega^*).$$

(c) The case of $kz=4/\omega_M$

In this case we have $w^\circ_{Max}=\frac{1}{2}\omega_M$, so

$$\omega_*=2\omega^*$$

Then eq. (111) gives

$$\left[K_z^*(\omega^*)\right]_{kz=4/\omega M} = CW_{0,M}\left(\frac{4}{\omega_M}\right)W_0^*(2\omega^*)\,K^*(\omega^*).$$

The characteristic curves of these three are shown in Fig. 15, where as the function $K(\omega)$ the characteristic function of the normal detection is chosen. But the curve of solid line is the curve of the normalized characteristic function $K^*_{1,3}(\omega^*)$. From Fig. 15 the characteristics of the wave number response in the present

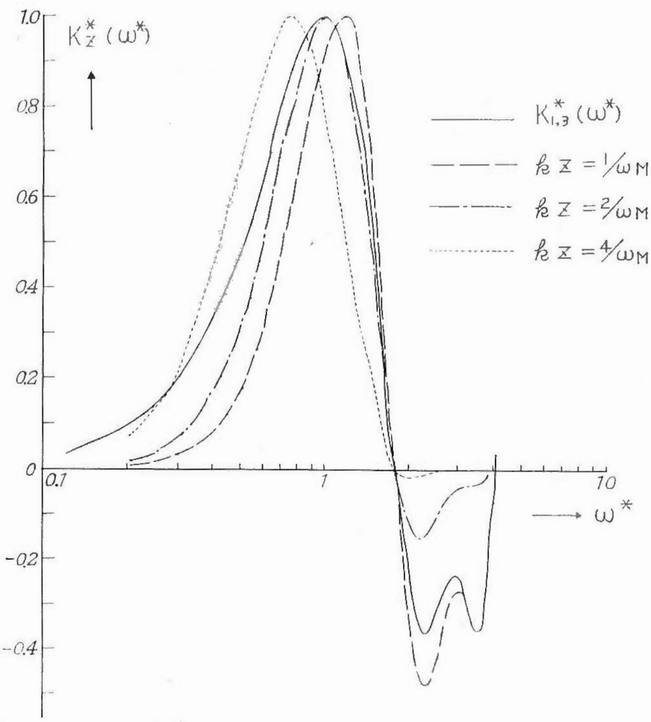

Fig. 15 Filter curves of $K_z^*(\omega^*)$ for several values of kz

case become clear as follows: That is, the characteristic function $K_z^*(\omega^*)$ at the depth $z=2/k\omega_M$ at which the central wave number of the normal detection becomes the same as the maximum detected wave number has the both characters of low cut and high cut slightly stronger than those of $K_{1,3}^*(\omega^*)$. And at the depth $z=1/k\omega_M$ which is the depth of maximum amplitude of a density variation of the same wave number as the central wave number of the normal detection the character of low cut of the characteristic function $K_z^*(\omega^*)$ becomes more remarkable compared with the former case, but in this case the influence of noises becomes larger. And the central wave number of the characteristic function $\left[K_z^*(\omega^*)\right]_{kz=1/\omega M}$ is nearly equal to 1/5S. For the case of $kz=4/\omega_M$ the central wave number of the characteristic function $\left[K_z^*(\omega^*)\right]_{kz=4/\omega M}$ transit to 1/7.6S, and the character of high cut becomes remarkable as shown in Fig. 15 then the influence of noise is not almost present in this depth.

From the consideration mentioned above, it is supposed that the residual density distribution which contribute substantially to the residual gravity may have similar figure to that of the residual gravity distribution at arbitrary depth.

If the characteristic function $K_3{}^a(\omega)$ is used as the function $K(\omega)$, the combined characteristic function $K_z(\omega)$ is expressed by the simple representation of

$$K_z(\omega)=\frac{4}{3}(1+k)^2\frac{\omega}{a}\sin^3 a\omega e^{-k|\omega|z}.$$

The result of the consideration of the characteristic of the above combined characteristic function is the very same as mentioned above.

VI. 3 Numerical Calculation of Residual Density

VI. 3. 1 Analytical Representation to calculate Residual Densities

In the equation

$$\Delta\rho(x,z)=\frac{1-e^{-wz}}{2\pi\gamma w}\int_{-\infty}^{\infty}(1+k)^2\omega^2 e^{-k|\omega|z}\Delta G(\omega)e^{i\omega x}d\omega$$

we can make

$$\omega^2 e^{-k|\omega|z}=\frac{1}{k^2}\frac{\partial^2}{\partial z^2}e^{-k|\omega|z}.$$

Then we have

$$\Delta\rho(x,z)=\frac{1-e^{-wz}}{2\pi\gamma w}\omega\frac{(1+k)^2}{k^2}\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty}e^{-k|\omega|z}\Delta G(\omega)e^{i\omega x}d\omega.$$

The function defined by

$$\mathcal{G}(x,z)=2\pi\gamma\int_{-\infty}^{\infty}e^{-k|\omega|z}\Delta G(\omega)e^{iwx}d\omega \tag{112}$$

can be considered as that this expression means a gravity distribution on the earth surface when the condensation surface on which a spectrum of a density distribution is denoted by $\Delta G(\omega)$ is considered. By using this "transformed residual gravity" $\mathcal{G}(x,z)$, we can have the analytical representation of a residual density. That is,

$$\Delta\rho(x,z)=\frac{(1+k)^2(1-e^{-z/2})}{2(\pi\gamma k)^2}\frac{\partial^2}{\partial z^2}\mathcal{G}(x,z). \tag{113}$$

Similarly, for eq. (86) we have

$$\Delta\rho(x,z)=\frac{(1+k)^2(1-e^{-z/2})}{2(\pi\gamma k)^2}\left[\frac{\partial^2}{\partial z^2}\mathcal{G}(x,z)-\frac{k}{2(1+k)}\frac{\partial}{\partial z}\mathcal{G}(x,z)\right]. \tag{114}$$

Moreover, when α can not be neglected, i. e., for eq. (85), we have

$$\Delta\rho(x,z)=\frac{(1+k)^2(1-e^{-(z+\alpha)/2})}{2(\pi\gamma k)^2}\left[\frac{\partial^2}{\partial z^2}\mathcal{G}_a(x,z)-\frac{k}{2(1+k)}\frac{\partial}{\partial z}\mathcal{G}_a(x,z)\right], \tag{115}$$

where

$$\mathcal{G}_a(x,z)=2\pi\gamma\int_{-\infty}^{\infty}e^{-k|\omega|z}\frac{\Delta G(\omega)}{1+\alpha(1+k)|\omega|}e^{i\omega x}d\omega. \tag{116}$$

VI. 3. 2 Computing Formulae

In order to calculate a residual density, eq. (113) or eq. (114) is used according to a scale of a residual gravity. Then at first the computing formula of the case of eq. (113) will be reduced.

Now, in order to calculate the transformed residual gravity eq. (112) is deformed as follows:

$$\begin{aligned}\mathcal{G}(x,z)&=2\pi\gamma\int_{-\infty}^{\infty}e^{-k|\omega|z}e^{i\omega x}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\Delta g(\xi)e^{-i\omega\xi}d\xi\right]d\omega\\&=\gamma\int_{-\infty}^{\infty}\Delta g(\xi)\left[\int_{-\infty}^{\infty}e^{-k|\omega|z}e^{i\omega(x-\xi)}d\omega\right]d\xi.\end{aligned} \tag{117}$$

In the above equation, if the discrete values of residual gravity with the uniform spacing S are given, the continuous function $\Delta\tilde{g}(x)$ which is the approximate function of the true function $\Delta g(x)$ and does not include a variation of its wave length below $\lambda_S=2S$ is expressed by the following equation as given by Tomoda and Senshu[23].

$$\varDelta\tilde{\tilde{g}}(\xi)=\sum\varDelta g(nS)\frac{\sin\frac{2\pi}{\lambda_S}(\xi-nS)}{\frac{2\pi}{\lambda_S}(\xi-nS)}. \tag{118}$$

Accordingly, the corresponding transformed residual gravity is given by the Tomoda and Senshu's method as

$$\mathscr{G}(x,z)=\gamma\sum\varDelta g(nS)\,\tilde{\tilde{\phi}}(x-nS,\ z) \tag{119}$$

where $\tilde{\tilde{\phi}}(x,z)$ is the density response for the unit gravity $\tilde{\tilde{\delta}}(x)$, namely, the unit gravity $\tilde{\tilde{\delta}}(x)$ is $\tilde{\tilde{\delta}}$ given by

$$\tilde{\tilde{\delta}}(x)=\frac{\sin\frac{2\pi}{\lambda_S}x}{\frac{2\pi}{\lambda_S}x}, \tag{120}$$

then $\tilde{\tilde{\phi}}(x,z)$ is expressed by

$$\tilde{\tilde{\phi}}(x,z)=\frac{\lambda_S}{4\pi}\int_{-\frac{2\pi}{\lambda_S}}^{\frac{2\pi}{\lambda_S}}e^{-k|\omega|z}\,e^{i\omega x}\,d\omega. \tag{121}$$

Therefore, we have

$$\tilde{\tilde{\phi}}(x,z)=\frac{\lambda_S}{2\pi}\left\{\frac{kz}{x^2+k^2z^2}+\frac{x\sin\frac{2\pi}{\lambda_S}x-kz\cos\frac{2\pi}{\lambda_S}x}{x^2+k^2z^2}e^{-\frac{2\pi}{\lambda_S}kz}\right\}. \tag{122}$$

This equation is also expressed by

$$\tilde{\tilde{\phi}}(x^*,z^*)=\frac{1}{\pi}\left\{\frac{z^*}{x^{*2}+z^{*2}}+\frac{x^*\sin\pi x^*-z^*\cos\pi x^*}{x^{*2}+z^{*2}}e^{-\pi z^*}\right\}, \tag{122$'$}$$

where $x^*=x/S$ and $z^*=kz/S$.

Thus eq. (119) becomes

$$\mathscr{G}(x^*,z^*)=\gamma\sum\varDelta g_n\,\tilde{\tilde{\phi}}(x^*-n,z^*). \tag{123}$$

The function $\mathscr{G}(x,z)$ can be used instead of $\mathscr{G}(x,z)$, then we have the following equation to calculate a residual density.

$$\varDelta\tilde{\tilde{\rho}}(x^*,z^*)=\frac{(1+k)^2}{2(\pi\gamma)^2}(1-e^{-z/2})\frac{1}{S^2}\frac{\partial^2}{\partial z^{*2}}\mathscr{G}(x^*,z^*). \tag{124}$$

Eq. (122)$'$, eq. (123) and eq. (124) give the following computing formula of a residual density at arbitrary subsurface point.

$$\begin{aligned}\varDelta\tilde{\tilde{\rho}}(x^*,z^*)=\frac{(1+k)^2}{2\pi^3\gamma S^2}(1-e^{-z/2})\sum\varDelta gn\Big[2z^*\frac{z^{*2}-3(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^3}-\\ -\frac{\pi^2z^*\cos\pi(x^*-n)+2\pi\cos\pi(x^*-n)-\pi^2(x^*-n)\times}{z^{*2}+(x^*-n)^2}\\ \frac{\times\sin\pi(x^*-n)}{}e^{-\pi z^*}\\ -\frac{z^*\{2\cos\pi(x^*-n)-4\pi(x^*-n)\sin\pi(x^*-n)\}-}{\{z^{*2}+(x^*-n)^2\}^2}\\ \frac{-6(x^*-n)\sin\pi(x^*-n)+4\pi(x^*-n)^2\cos\pi(x^*-n)}{}e^{-\pi z^*}\\ +\frac{8z(x^*-n)^2\cos\pi(x^*-n)-8(x^*-n)^3\sin\pi(x^*-n)}{\{z^{*2}+(x^*-n\}^2\}^3}e^{-\pi z^*}.\end{aligned} \tag{125}$$

At each measurement point eq. (125) becomes simpler as under equation:

$$\begin{aligned}\varDelta\tilde{\tilde{\rho}}(x^*z^*)=\frac{(1+k)^2}{\pi^3\gamma S^2}(1-e^{-z/2})\sum\varDelta g_n\Big[z^*\frac{z^{*2}-3(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^3}-\\ -(-1)^{x^*-n}\frac{\pi(1+\frac{\pi}{2}z^*)}{z^{*2}+(x^*-n)^2}e^{-\pi z^*}-(-1)^{x^*-n}\frac{z^*-2\pi(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^2}e^{-\pi z^*}+\end{aligned}$$

$$+(-1)^{x^*-n}\frac{4z^*(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^3}e^{-\pi z^*}\Big]. \qquad (126)$$

By using eq. (126) a residual density distribution can be calculated very easily from a residual gravity distribution when the values of inner part of the large bracket in eq.(126) are tabulated for the values of $m=|(x^*-n)|$. Table 5 is such a table for several values of z.

Table 5 Values of the inner part of the bracket in eq. (126) for several values of z*

m	$z^*=0.5$	$z^*=1.0$	$z^*=2.0$
0	1.6730	0.6078	0.1184
1	-0.7535	-0.1542	0.0206
2	-0.0460	-0.1106	-0.0336
3	-0.0316	-0.0167	-0.0195
4	0.0029	-0.0147	-0.0117
5	-0.0079	-0.0011	-0.0056
6	0.0031	-0.0043	-0.0036
7	-0.0034	0.0004	-0.0015
8	0.0018	-0.0019	-0.0014
9	-0.0019	0.0006	-0.0006
10	0.0013	-0.0011	-0.0006
11	-0.0011	0.0004	-0.0003
12	0.0010	-0.0006	-0.0004

In many cases eq.(126) can be used approximately for the purpose of obtaining a residual density, but we can neither use eq. (126) for the case of a residual gravity of large scale nor for the regional gravity. In such a case the exact computing formula reduced from eq. (115) must be used. As the result of similar reduction, we have

$$\Delta\tilde{\bar{\rho}}(x^*, z^*)=\frac{(1+k)^2}{\pi^3\gamma S^2}(1-e^{-z/2})\sum\Delta g_n\Big[z^*\frac{z^{*2}-3(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^3}-$$
$$-(-1)^{x^*-n}\frac{\pi(1+\frac{\pi}{2}z^*)}{z^*+(x^*-n)^2}e^{-\pi z^*}-(-1)^{x^*-n}\frac{z^*-2\pi(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^2}e^{-\pi z^*}+$$
$$+(-1)^{x^*-n}\frac{4z^*(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^3}e^{-\pi z^*}+\frac{S}{4(1+k)}\Big\{\frac{z^{*2}-(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^2}-$$
$$-(-1)^{x^*-n}\frac{1+\pi z^*}{z^{*2}+(x^*-n)^2}e^{-\pi z^*}+(-1)^{x^*-n}\frac{2(x^*-n)^2}{\{z^{*2}+(x^*-n)^2\}^2}e^{-\pi z^*}\Big\}\Big]. \qquad (127)$$

VII. Application of the Present Method and its Discussions

VII. 1 Numerical Example

In Fig. 16, the Bouguer anomaly on a measurement line observed in Akita Plane is shown. And the residual gravities calculated by the running average method are shown in Fig. 17. The curve of solid line in Fig. 17 indicates the variation of the normal structure and the curve of dashed line indicates the variation of the bi-structure with the uniform spacing S=250m. Moreover, Fig. 18 (a), (b) and (c) show the corresponding residual densities at several depths for several values of k. And the curves of solid line in these figures indicate the normal

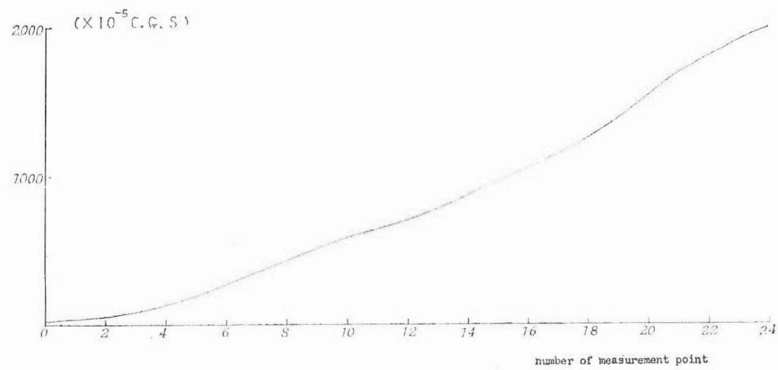

Fig. 16 Bouguer anomaly observed in Akita plane (Spacing S=500m)

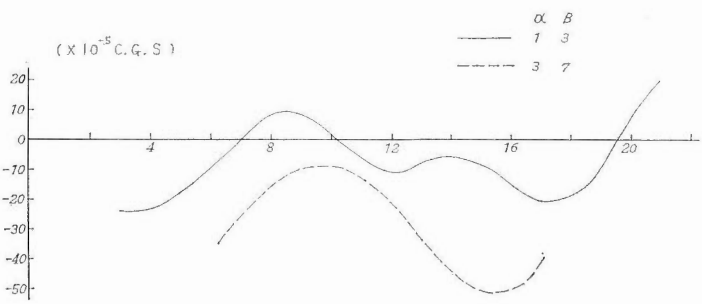

Fig. 17 Residual gravities obtained by the present method from the data indicated in **Fig.** 16

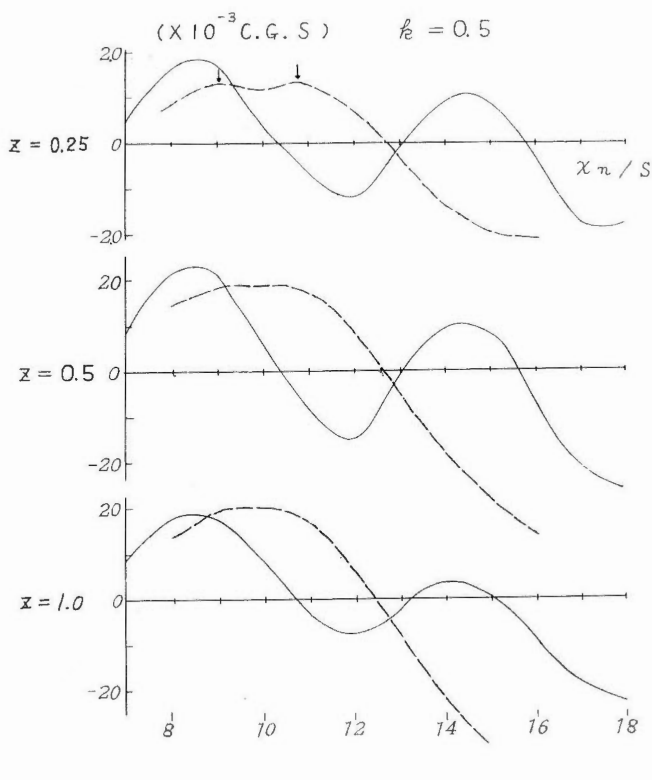

Fig. 18-a Residual densities at several depths (k=0.5)

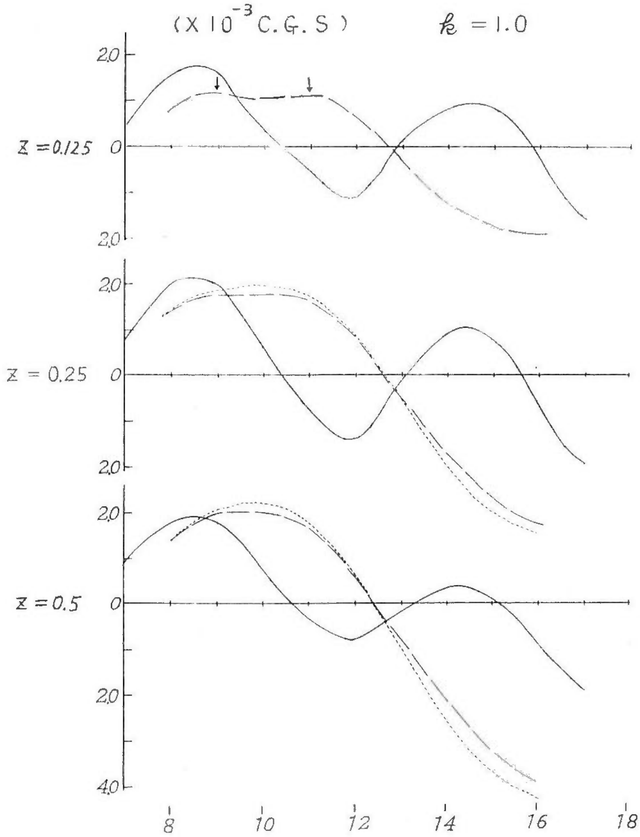

Fig. 18-b ditto (k=1.0)

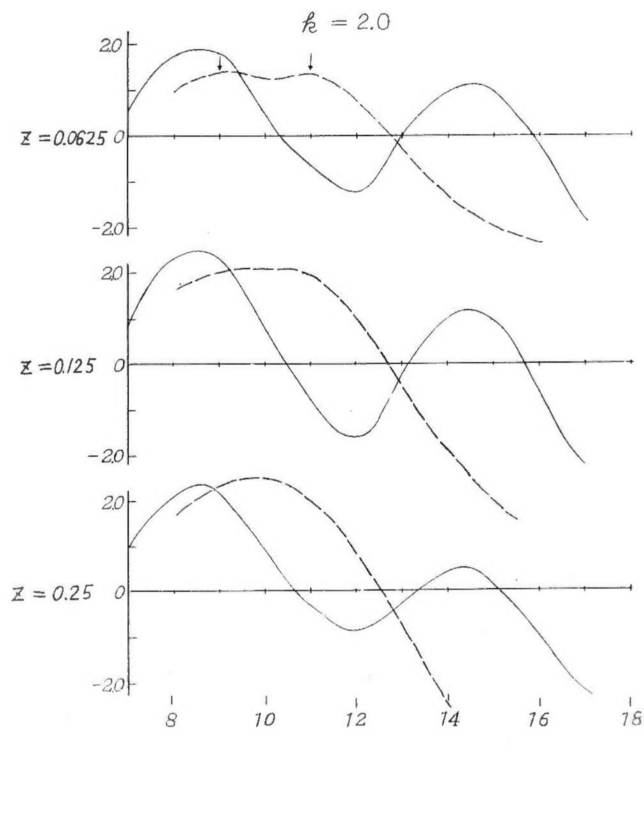

Fig. 18-c ditto (k=2.0)

structure and the curves of dashed line indicate the bi-structure. The results obtained from these figures are as follows:

(1) The curves of the residual density obtained are similar to the curves of the corresponding residual gravity in wide range of depth.

(2) The residual density obtained has the maximum amplitude at about $kz=S$ ($=0.25$) for the normal structure.

(3) The amplitude of the residual density variation vary its magnitude very gradually with depth.

(4) The residual density does not vary extremely its shape and its magnitude for different value of k but the same value of kz.

These natures of the curves of the residual density are already expected in the previous discussions. Moreover, the following natures are recognized for the bi-structure of the density.

(5) The two parts with the arrow head of each curve for $kz=S/2$ ($=0.125$) in the figures indicate the influences of the variations of small scale and these become disappear at deep depth.

(6) The curves of dotted line in Fig. 18(b), which are the curves calculated by the exact computing formula eq. (127), are nearly same as the curves of dashed line which are calculated by the approximate formula eq. (126), i. e., by the use of Table 5.

These matters are also already expected in the previous chapter.

VII. 2 On the Interpretation of Residual Densities

In the interpretation of residual densities it becomes very important that

(1) what value of k should be taken

and

(2) how we should interpret the normal structure and the bi-structure.

In the following, these two subjects will be considered.

For the question (1) there is no answer, that is, any theoretical foundation and any empirical foundation are not present for the selection of the value of k. However, when the fact that the depth of the presence of the residual density becomes shallow with large value of k is considered, the following presumption will be obtained. That is, the parameter k depends on the dynamical characteristics of the force by which the objective subterranean structures are formed, the depth of action of this force, the period of the tectogenesis, and on the history and the natures of the sedimentary formations etc. When a weak force acted on sedimentary formations consolidated sufficiently, deformations of formations and variations of their densities might be little. Accordingly, an influence to a gravity value on the earth surface may be small and geological structures produced may be of small scale. On the contrary, when the consolidation of rocks is not sufficient, large variations may produce in formations even for small force. So in this case geological structures produced may appear comparatively remarkable in the considerable range of depth. From such a consideration it may be considered that for sufficiently consolidated formations a large value of k should be taken and for unconsolidated formations a small value of k should be taken. Here, we must pay attention to that influences of tectonic forces to sedimentary formations become perfectly different each other even when forces of same magnitude and of same scale acted on same formations if periods of actions of tectonic forces are different each other. Then a value of k may become different in each case. Generally speaking, shallow formations are in the unconsolidated state, so in this case a small value should be taken as a value of k. Although, for the various geological structures in deep formations we can not say as mentioned above. Because, it is supposed that deep formations are in the consolidated state owing to the action of compaction under very high pressure caused by upper thick formations in the presence, but formations at deep depth may be in the unconsolidated state in the past when the geological structures were formed.

In the present stage the numerical calculations of the residual density should be performed for $k=0.5, 1.0$ and 2.0 as done in the previous section. And then if there are many or few data, e. g., geological, bore hole and seismic data and so on, a residual density should be calculated by assuming a suitable value of k. Moreover, in this time it may be possible that the value of k assumed for the normal structure is different from that for the bi-structure.

If such a consideration is admitted, the quantitative interpretation of the residual gravity becomes more difficult and complicated. Then the relation between the normal structure and the bi-structure of density will be considered in accordance with the example of Fig. 18.

As shown in Fig. 18, we have the positive residual densities for both the normal structure and the bi-structure on the Tsuchizaki anticline. Then it is presumed that this anticline grow vertically in the considerable range of depth. On the other hand, we have the positive anomaly only for the normal structure and the negative anomaly for the bi-structure on the Yabase anticline. Then it is presumed that this anticline may grow in small scale in shallow formations.

Such a presumption as mentioned above also holds good qualitatively even when different values of k are taken for the respective structures. However, in this case the subterranean structures become differ from considerably those of the case in where the same value of k is assumed for the both structures. This matter will be evident through the following argument.

Now suppose

$$\tilde{\rho}(x, z)=\tilde{\rho}_N(x, z)+\tilde{\rho}_B(x, z),$$

where

$$\tilde{\rho}_N(x, z)=\rho_2(z)\cdot\rho_N(x, z),$$
$$\tilde{\rho}_B(x, z)=\rho_2(z)\cdot\rho_B(x, z)$$

in eq. (72). Then the following relations are assumed.

$$\Delta_N\tilde{\rho}(x, z)=\Delta_N\tilde{\rho}_N(x, z),$$
$$\Delta_B\tilde{\rho}(x, z)=\Delta_B\tilde{\rho}_B(x, z),$$

where $\Delta_N=\Delta_{1,3}$, i. e., the operator of the normal detection, and $\Delta_B=\Delta_{3,7}$, i. e., the operator of the bi-structural detection. Further, the following relations are assumed.

$$\nabla^2_{k_N}\rho_N(x, z)\equiv\left[\frac{\partial^2}{\partial x^2}+\frac{1}{k_N^2}\frac{\partial^2}{\partial z^2}\right]\rho_N(x, z)=0,$$
$$\nabla^2_{k_B}\rho_B(x, z)\equiv\left[\frac{\partial^2}{\partial x^2}+\frac{1}{k_B^2}\frac{\partial^2}{\partial z^2}\right]\rho_B(x, z)=0.$$

By these assumptions we have $\rho_2(z)\cdot\Delta_N\rho_N(x, z)$ as the residual density corresponding to the normal structure of gravity, and also have the residual density $\rho_2(z)\cdot\Delta_B\rho_B(x, z)$ corresponding to the bi-structure of gravity.

In the following the residual densities obtained for various values of k and k will be considered.

(a) The case of $k_N=1.0$ and $k_B=0.5$

From Fig. 18 (a) and (b) we can see easily that the normal structure of density the maximum amplitude at the depth about $z=0.25$ (km) and the bi-structure of density has the maximum amplitude at the depth about $z=1.0$ (km). And their amplitudes vary very gradually in considerable ranges of depth. From these two figures it is presumed that the Tsuchizaki anticline may grow in formations at depth shallower than a few kilometers and on the other hand, the Yabase anticline may grow in shallow formations at depth within several hundreds meters. Fig. 19 shows the residual density composed of the normal structures and the bi-structure, and the curves of solid line represent the normal structure for $k_N=1.0$ and the curves with the simbol $C_{0.5}$ of dashed line indicate the bi-structure for $k_B=0.5$. Moreover, the curves with the symbol $C_{1.0}$ of dotted line represent the bi-structure for $k_B=1.0$. And the curves of dashed line of the other curves represent the residual density composed of the normal structure and the bi-structure $C_{0.5}$, and the curves of dotted line with no symbol represent the residual density composed of the normal structure and the bi-structure $C_{1.0}$. The latter curves are those of composed residual density $\Delta_{1,7}\rho(x,z)$ for $k=1.0$ corresponding to the composed residual gravity

$$\Delta_{1,7}\,g(x)=\Delta_{1,3}\,g(x)+\Delta_{3,7}\,g(x).$$

From Fig. 19 it is recognized that the difference between $\Delta_{1,7}\rho(x,z)$ for $k=1.0$ and the residual density composed of $\Delta_N\rho(x,z)$ for $k=1.0$ and $\Delta_B\rho(x,z)$ for $k=0.5$ is not large but in the latter the double structure with depth in Yabase district

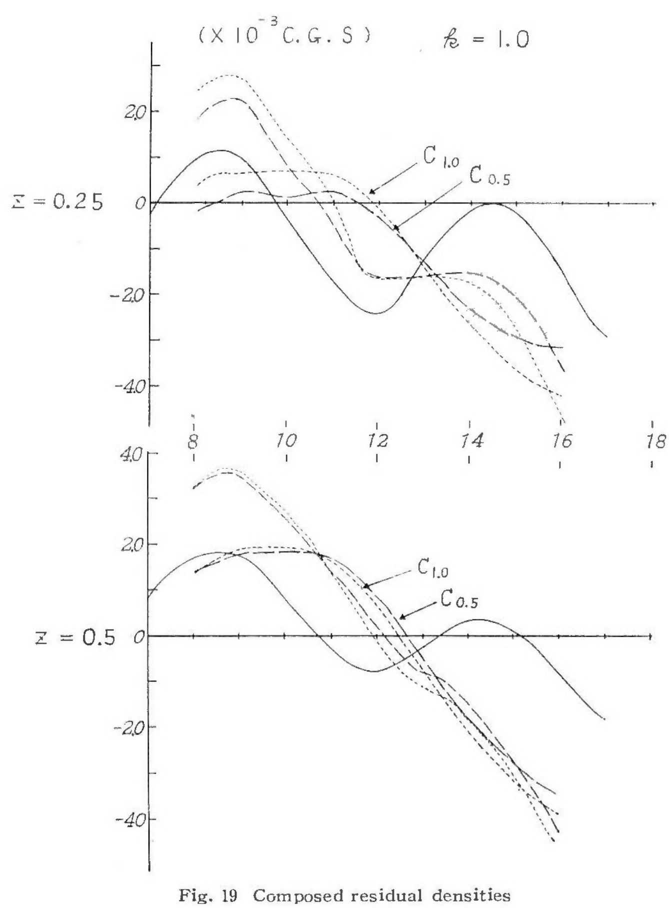

Fig. 19 Composed residual densities

becomes somewhat remarkable. This is the matter already expected from Fig. 18.

(b) The case of $k_N=2.0$ and $k_B=0.5$

In this case the normal structure indicating the Yabase anticline becomes present in very shallow range within a few hundreds meters. And the double structure with depth in the Yabase district becomes remarkable. Then if the existence of the Yabase anticline could not be known, the anomaly indicating the Yabase anticline will not attract our notice as far as the present case is considered.

(c) The case of $k_N=0.5$ and $k_B=1.0$

In this case the Yabase anticline grow in considerable range of depth within about 1 km. And the residual density composed of the both structures gives us similar view as $\Delta_{1,7}\rho(x,z)$ for $k=0.5$ (not illustrated).

(d) The case of $k_N=0.5$ and $k_B=2.0$

In this case, from Fig. 18 (a) and (b), it is presumed that there are two anticlines, i.e., one is strong and the other is weak, in the range of depth within about 1 km. And the double structure with depth in the Yabase district becomes disappear, so it is presumed that the normal structure and the bi-structure may be caused by the same subterranean structure.

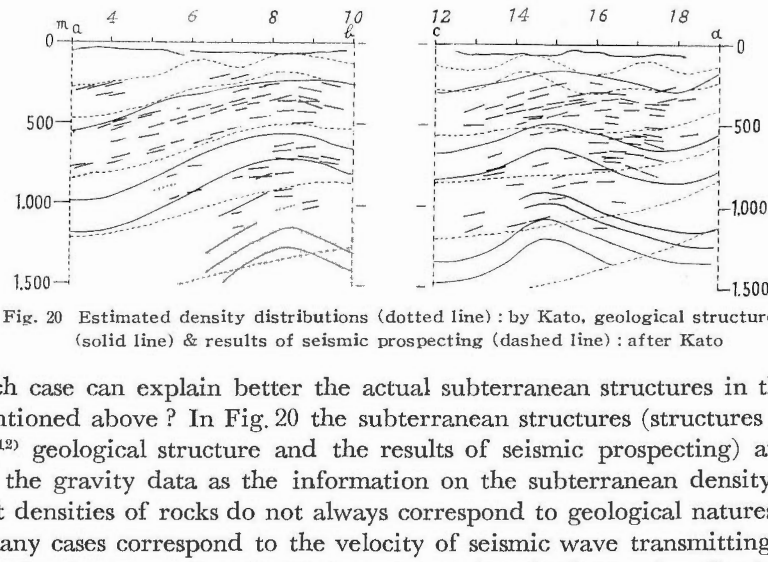

Fig. 20 Estimated density distributions (dotted line) : by Kato, geological structure (solid line) & results of seismic prospecting (dashed line) : after Kato

Which case can explain better the actual subterranean structures in the several cases mentioned above ? In Fig. 20 the subterranean structures (structures estimated by Kato,[12] geological structure and the results of seismic prospecting) are shown. There is the gravity data as the information on the subterranean density distribution. But densities of rocks do not always correspond to geological natures of rocks, but in many cases correspond to the velocity of seismic wave transmitting in rocks. Then the density anomalies should be compared with the results of seismic prospecting. In Fig. 20 it is recognized that there are two anticlines, one of which grows in considerable range of depth in the Tsuchizaki district, i.e., this is the Tsuchizaki anticline, and the other grows in small scale at the depth within 1 km, i.e., the Yabase anticline. So the case (c) of the cases mentioned above is most suitable to the results of seismic prospecting. Then in this example the case of $k_N=0.5$ and $k_B=1.0$ or the case of $k_N=0.5$ and $k_B=0.5$ is suitable. This conclusion proves the previous consideration that for shallow formations of unconsolidated state a small value should be taken as a value of k.

VII. 3 On the Interpretation of Gravity Anomaly

Gravity anomalies observed are, as already often mentioned, caused by various subterranean structures of large and small scale at various depths. Then we should consider not only the normal structure and the bi-structure, but also anomalies of other scale, particularly regional gravity, in an interpretation of gravity anomalies. For the purpose of an interpretation of a regional gravity eq. (127) or the trial and error method. In the use of eq. (127) it becomes also important how we should choose a value of k. And the trial and error method is applicable when there is a large difference between the density of the bedrock and the average density of the upper formations. For the purpose of the application of this method, the standard curves which are the gravity anomaly curves caused by assumed basement with the various simple geometrical shapes are now being made in our Geological Survey.

Then it is concluded that an interpretation of gravity anomalies should be performed according to the following procedure.

(a) To calculate the residual gravities of three kinds, those are the noise structure, the normal structure and the bi-structure, and the regional gravity.

(b) To calculate the residual densities corresponding to respective residual gravities for the various values of the parameter k, e. g., $k=0.5$, 1.0 and 2.0, at several depths, e. g., $z=0.5\,S/k$, S/k and $2S/k$.

(c) To consider the composed residual densities as seen in the previous section.

(d) To estimate the regional tendency of geological formations or of the basement by means of the present method or of the trial and error method (or standard curves) respectively.

(e) To presume the subterranean structures from the results of the considerations in the processes (c) and (d).

VIII. Summary and Conclusion

In the present paper at first the writer discusses the filtering effects of the running average method, those are in chapter II one dimensional case and in chapter III two dimensional case. As the results of these investigations he obtains the following results.

(1) It is found that $K_3^a(\omega)$ is the most excellent characteristic function of the three, i. e., $K_2^a(\omega)$, $K_3^a(\omega)$ and $K_4^a(\omega)$, of the extended running average method.

(2) Characteristic function $K_{\alpha,\beta}(\omega)$ of a case in which the ratio $(2\beta+1)/(2\alpha+1)$ has a value of about 2, e. g., $K_{1,3}(\omega)$, $K_{3,7}(\omega)$, etc., can be approximated by $K_2^a(\omega)$, and a characteristic function $K_{\alpha,\beta}(\omega)$ possessing a ratio $(2\beta+1)/(2\alpha+1)\sim 3$ can be approximated by $K_3^a(\omega)$.

(3) In the two dimensional case, it is found that $\overset{\text{II}}{\underset{2.5a}{K}}(\lambda)$ is the most excellent selection coefficient of the three, i. e., $\overset{\text{II}}{\underset{2a}{K}}(\lambda)$, $\overset{\text{II}}{\underset{3a}{K}}(\lambda)$ and $\overset{\text{II}}{\underset{2.5a}{K}}(\lambda)$.

(4) It is concluded that $\overset{\text{II}}{\underset{2a}{K}}(\lambda)$ is adoptable as the approximate selection coefficient of the two dimensional case, when the theoretical consideration of gravity anomalies detected by the present method is done.

In the next place in chapter IV the physical meanings of the present detections and the quantities detected by these detections are considered in one dimensional case. And the following result is obtained.

(5) When the physical quantities $g(x)$ are caused by the distribution of the physical quantities $\rho(x,z)$ and if the functional form of $\rho(x,z)$ is known, the physical quantity which correspond to the residual quantity $\Delta g(x)$ at arbitrary value of the variable z is the residual quantity $\Delta\rho(x,z)$, and the quantity $\Delta\rho(x,z)$ can be determined uniquely from the distribution of the residual quantity $\Delta g(x)$.

In chapter V the reasons by which the normal detection and the bi-structural detection are used in gravity prospecting are mentioned.

Further, the writer studies the quantitative interpretation of the residual gravity obtained by his method. The results of this study are mentioned in detail in chapter VI. In this study he presumes the two assumptions on the density distribution. And he considers at first the characteristics of the density-spacial filter. The results obtained are as follows:

(6) A density variation of arbitrary wave number has the maximum amplitude at the depth z_M determined by its wave number. And if ω is large, the following relation is obtained.

$$z_M \fallingdotseq \frac{1}{k\omega}.$$

(7) A density variation of an arbitrary wave number changes its amplitude with depth very slowly in the considerable range of depth about the depth of maximum amplitude z_M.

(8) The characteristic curve of the density-spacial filter does not change its shape with depth in relative expression. And the central wave number of this filter, namely, the maximum detected wave number, at arbitrary depth z is determined by its depth. Then if a density variation is considered at shallow depth, the following relation is obtained.

$$\omega^0{}_{\mathrm{Max}} = \frac{2}{kz}.$$

(9) It is supposed that the distribution of the residual density indicates the similar figure of distribution to the distribution of the residual gravity in the range of depth about the depth of maximum amplitude z_M.

In VI. 3 the convenient computing formulae for a residual density are reduced.

Finally, in chapter VII the considerations on the interpretation of gravity anomalies are mentioned.

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重力探査における新解析法について

瀬谷 清

要 旨

われわれの行なう重力探査においては，基盤調査以外に種々の深度における大小の規模の地下構造の存在を推定することをその目的とすることが多く，したがって，重力探査における解析法としては局部的な一般には微弱な異常の検出，およびその検出された異常の解釈は重大な意義を有している。

本論文では余剰重力を求めるために提案された筆者の方法（移動平均法）および余剰重力の定量的解釈に関する理論的考察が述べられている。

第II章では移動平均法の有する sampling filter としての解析的な性質が一次元的な場合について詳しく調べられ，第III章では二次元問題の場合が論じられている。その結果次のことがらが明らかにされた。

(1) “拡張された移動平均法”の3種の検出の有する特性函数 $K_2^{a}(\omega)$, $K_3^{a}(\omega)$, および $K_4^{a}(\omega)$のうち $K_3^{a}(\omega)$ が最も優れた特性を有する。

(2) 移動平均法の各種検出の有する特性函数 $K_{\alpha,\beta}(\omega)$ は比$(2\beta+1)/(2\alpha+1)$が2に近い値を有するときは $K_2^{a}(\omega)$ で近似され，その比が3に近い値を有するときは $K_3^{a}(\omega)$ で近似される。ゆえに，われわれが余剰重力を求めるために使用する検出：正規検出および倍構造検出：の特性函数は $K_2^{a}(\omega)$ でよく近似できることになる。このことは問題の理論的取り扱いが解析的に簡略に行なえることを意味している。

(3) 二次元の場合には，ある任意の点の異常量がその点を中心とする2つの円に関する平均量の差で定義される検出を考えるとき（(60) 式)，その最も優れた特性を有する選択係数は$K^{\mathrm{II}}_{2.5a}(\lambda)$であると思われる。

(4) 二次元の場合に拡張された移動平均法（正規および倍構造検出）の有する選択係数は簡単な解析表示（(63)式）を有する $K^{\mathrm{II}}_{2a}(\lambda)$ によって近似することができる。

第IV章では移動平均法の有する直接的な意味および検出された異常量の物理的な意味が考察されている。その結果

(5) 物理量$g(x)$が物理量 $\rho(x, z)$ の分布により決定され，そしてもし $\rho(x, z)$ の函数形が知られているときは，余剰量 $\Delta g(x)$ に対応する量は，任意の z の値に対し，$\Delta\rho(x,z)$でなあり，この $\Delta\rho(x, z)$ は $\Delta g(x)$ の分布より一意的に求めることができることが明らかとった。

このことは重力異常の解釈において，余剰重力 $\Delta g(x)$ に対応する密度異常：余剰密度：が任意の深さにおいて求められ得ることを示している。このとき地下の密度分布に適当な数学的模型を仮定せねばならない。

第V章では重力探査において正規検出および倍構造検出が用いられる理由が述べられている。

第VI章では第IV章の考察に基づき，地下の密度分布に (71) 式，(72) 式，(77) 式，および (78) 式の諸式で示される数学的密度分布模型を仮定することによって余剰重力の定量的解釈に関する理論的考察を行ない，その結果として余剰重力に対応する地下の余剰密度を算出する公式，(126) 式および (127) 式を求めている。なおここでの議論を通じて明らかとな

った重要な事項は次のとおりである。

(6) 地下における任意波数の密度変化はその波数によって決まる深さ z_M で最大振幅を有する。もしこのとき波数が比較的大ならば"最大振幅深度" z_M と波数との間には (90)′式で示されるきわめて簡単な重要な関係が得られる。

(7) 任意波数の地下の密度変化はその波数によってきまる最大振幅深度 z_M の前後のかなりの深度範囲でゆるやかに深度とともにその振幅が変化する。

(8) 仮定された函数形の密度分布と空間の存在が示す filter 効果が詳細に調べられ，その結果任意深度におけるこの"密度-空間 filter"の中心波数（"極大検出波数"）はその深度によってきまり，もし比較的浅い深度での密度変化を考えるならばきわめて簡単な関係式，(93) 式で与えられる。

(9) 筆者の方法によって得られる余剰密度分布は最大振幅深度の前後の深度において余剰重力分布の示す変化曲線に類似した変化曲線を示すことが推論される。このことは従来の定性的重力異常解釈の理論的根拠となる結論である。

最後に第Ⅶ章では重力異常の解釈についての考察が実例によって論ぜられている。

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- b．岩石・鉱物
- c．古生物
- d．火山・温泉
- e．地球物理
- f．地球化学

B．応用地質に関するもの
- a．鉱　床
- b．石　炭
- c．石油・天然ガス
- d．地下水
- e．農林地質・土木地質
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C. Miscellaneous

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Seya, K.

On the New Method of Analysis in Gravity Prospecting

Kiyoshi Seya

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28 illus., 5 tab.

In the present paper, the results of writer's study on "the Running Average Method" which has been proposed by him as the method to detect local and weak gravity anomalies and on the quantitative interpretation of residual gravities obtained by his method are described. About the former he considers in detail the filtering effects of the detections of the Running Average Method. And as to the latter, the method by which subterranean density anomalies("residual densities")were directly calculated from the residual gravities is mentioned.

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