

## **A Preliminary Study of Earthquake and Underground Water Level Data via Fourier Cosine Spectrogram**

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The earthquake and underground water level data are two typical time series discrete data which involve much complicated information. In general, the underground water level data involve discontinuous jumps while most earthquake data are continuous. This study employs a sophisticated strategy to deal with the continuous time series data. It involves the following steps: employ the iterative Gaussian smoothing to remove the non-sinusoidal part; use searching procedure and interpolation to identify the local maximum or minimum points around the two ends of the sinusoidal part and consider these two points as two new ends; redistribute the remaining sinusoidal part so that its total points is an integer power of 2; take an even function with respect to an end; perform the Fast Fourier Transform (FFT) of the expanded data string that will results in a Fourier cosine spectrum; determine the frequency resolution of the spectrogram and the Gaussian window width on the spectral domain; find the inverse FFT of the band-pass limited spectrum defined by the Gaussian window centered at a given frequency that will give the real part of the Fourier cosine spectrogram; use the Hilbert transform to find the amplitude of the corresponding real part; and finally scan all the frequency to find spectrograms of the real part and amplitude. As to the related theoretical background, it can be shown that by applying the Gaussian smoothing method to smooth a data string involving a non-periodic polynomial of degree  $N$ , the resulting high frequency part will involve a non-periodic polynomial of degree  $N-2$  [1]. Consequently, by repeatedly applying the same Gaussian smoothing method more than  $N/2$  times to smooth the remaining high frequency part at each step, the final high frequency part will not contain a polynomial [1,2]. Since the expanded sinusoidal part satisfies all the periodic condition at the two ends, the resulting Fourier cosine spectrum is an approximate parameter representation of the sinusoidal part such that the derivative of any order can be obtained by directly differentiating the discrete Fourier cosine expansion [1]. It can also be shown that, both Morlet and Gabor transforms of the sinusoidal part localized by a Gaussian window with a window width of  $d$  on the time domain for a specific frequency,  $f$ , are equivalent to the inverse FFT of a corresponding Gaussian window on the spectral domain centered at the frequency,  $f$ , and with a window width of  $1/d$  [3-6]. As a consequence, the evaluation of the Fourier cosine spectrogram needs not be perform the convolution integrals of both the Morlet and Gabor transforms [7] so as to avoid the numerical integration error and non-periodicity. Moreover, the inverse transform of the spectrogram can be done by manipulating the Fourier cosine spectrum directly. In Ref.[3-6], resulting spectrograms were shown to have better visibility than that employing the Morlet and Gabor transforms. Both Fourier cosine spectrum and spectrogram are employed to study the low frequency response of the underground water level and earthquake data in Taiwan. Since the proposed data analysis technique can not handle a data string involving discontinuous jumps which reflect the earthquake, several continuous underground water level data between two earthquakes were examined. The long waves with wavelength longer than the half day underground tides were removed

by the iterative Gaussian smoothing method. All the high frequency part's spectrograms are different that means these data must strongly relate to the local geological structure. Moreover, all of them show unsteady variation of many low frequency modes which reflect the complicated heterotaxy induced by the interaction between the post effect of the previous earthquake and the initiation of the upcoming earthquake. Two set of the earthquake data related to the Taiwan 921 earthquake were examined, one runs across the earthquake period and the other is about 9 hours ahead of the earthquake. The latter data string involves a small earthquake. In the silence periods just ahead of both earthquakes, there are 1 Hz mode and other long wave modes. Although the 1 Hz mode may be caused by the instrument, its amplitude variation and the existence of the harmonics show that these low frequency modes might have certain physical meaning. In this stage, the earthquake data were also examined by converting the low frequency zone into the range which can be sensed by human ear. Two frequency shift techniques are employed. One just speeds up the broadcast rate by shrinking the data length and the other performs the frequency shift numerically data but preserves the data length. The speed up method shows that their post earthquake periods have similar ground thunder effect. The length preserving version produces different sound characteristics in strong motion periods but similar characteristics in silence periods. These limited test show that the proposed spectrogram and technique of converting data into sound are potential techniques for the future study in this field.

### References

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